

# **Essays in Financial Intermediation**

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# Declaration

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# Abstract

The thesis consists of three papers.

*Credit Rating and Competition* (co-authored with Pragyan Deb and Nelson Camanho) studies the behaviour of credit rating agencies in a competitive framework with the presence of conflicts of interest. We show that competition for market share through reputation is insufficient to discipline rating agencies in equilibrium. More importantly, our results suggest that, in most cases, competition will aggravate the lax behaviour of rating agencies, resulting in greater ratings inflation. This result has important policy implications since it suggests that enhanced competition in the ratings industry is likely to make the situation worse.

*Credit Default Swaps - Default Risk, Counter-party Risk and Systemic Risk* examines the implications of CDS on systemic risk. I show that CDS can contribute to systemic risk in two ways: through counter-party risk and through sharing of default risks. A central clearing house, which can only reduce counter-party risk, is by no means a panacea. More importantly, excessive risk taken by one reckless institution may spread to the entire financial system via the CDS market. This could potentially explain the US government's decision to bail out AIG during the recent financial crisis. Policies requiring regulatory disclosure of CDS trades would be desirable.

*Investor Cash Flow and Mutual Fund Behaviour* (co-authored with Zhigang Qiu) analyzes the trading incentives of mutual fund managers. In open-ended funds, investors are only willing to invest in the fund when the share price of the fund is expected to increase, i.e. the fund is expected to make profits in the future. We show that the fund manager may buy the asset even when he perceives the asset to be over-valued, given that his portfolio choices are disclosed to the investors and that he is paid a fixed fraction of the terminal value of the fund.

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# Credit Rating and Competition

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## Abstract

In principle, credit rating agencies are supposed to be impartial observers that bridge the gap between private information of issuers and the information available to the wider pool of investors. However, since the 1970s, rating agencies have relied on an issuer-pay model, creating a conflict of interest - the largest source of income for the rating agencies are the fees paid by the issuers the rating agencies are supposed to impartially rate. In this paper, we explore the trade-off between reputation and fees and find that relative to monopoly, rating agencies are more prone to inflate ratings under competition, resulting in lower expected welfare. Our results suggest that more competition by itself is undesirable under the current issuer-pay model and will do little to resolve the conflict of interest problem.

**Keywords:** Rating Agency, conflicts of interest, competition, reputation, repeated games, financial regulation

**JEL Classifications:** C73, D43, D82, D83, G24

# 1 Introduction

The credit rating industry aims to offer investors valuable information about issuers in need of financing. Due to the asymmetric information between the issuers and the investors, credit ratings often have pivotal impacts on the issuers' financing outcomes. Before the 1970s, the rating agencies relied on an investor-pay model wherein investors subscribed to ratings released by the agencies and these subscription revenues were the main source of income for the rating agencies. However owing to the 'public good' nature of ratings<sup>1</sup> and the increase in free riding, rating agencies switched to the current issuer-pay model and started charging issuers for ratings. As things stand today, the largest source of income for the rating agencies<sup>2</sup> are the fees paid by the issuers the rating agencies are supposed to impartially rate.<sup>3</sup> This tempts rating agencies to rate better than what fundamentals suggest, as many have pointed out during the recent sub-prime crisis.

It is often suggested that introducing more competition between rating agencies would alleviate the conflicts of interest. We develop an infinite horizon model where rating agencies compete for market share and face a trade-off between reputation and current fess. Competition in our model has two effects - the disciplining effect and the market-sharing effect. Competition decreases ratings inflation through the disciplining effect as rating agencies have incentives to maintain or gain the market leader position. On the other hand, the reward from maintaining reputation is lower because competition implies that the market is shared between a larger number of rating agencies, which we call the market-sharing effect. Our results suggest that in general the market-sharing effect will dominate and competition will aggravate ratings inflation and reduce expected welfare.

Given the structure of the market - with Standard & Poor's and Moody's having 80% of

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<sup>1</sup>This was officially recognised by the Securities and Exchange Commission (SEC) in the 1970s when the big three rating agencies – Standard & Poor's, Moody's and Fitch were designated self-regulatory entities. See Lowenstein (2008).

<sup>2</sup>It is also interesting to note that rating agencies are some of the most profitable businesses. Moody's has been the third most-profitable company in the S&P 500-stock index from 2002 to 2007, based on pretax margins (ahead of both Microsoft and Google).

<sup>3</sup>Summary Report of Issues Identified in the Commission Staffs Examinations of Select Credit Rating Agencies by the Staff of the Securities and Exchange Commission, 2008, p.9.

market share,<sup>4</sup> we model competition amongst the rating agencies in a duopolistic setting. In our model, issuers need a good rating to finance their projects. Rating agencies, which can be of two types - *honest* or *strategic*, perfectly observe the quality of the project and can either give the issuer a good rating or refuse rating. An honest rating agency always gives good ratings to good projects and no rating to bad projects while a strategic rating agency acts to maximise its expected profits. Neither investors nor issuers know for sure if a rating agency is honest and they Bayesian update on the reputation of the rating agencies (*i.e.* the probability that a rating agency is honest). The market share of the rating agency is modelled such that rating agencies with higher reputation will attract more projects. Hence the rating agencies face a trade-off between current income and reputation which determines their future market share and income.

We compare the behaviour of rating agencies between the duopolistic case and the monopolistic case<sup>5</sup> and find that on average rating agencies inflate ratings more under duopoly. Intuitively, given that the total market size is fixed, more competition will result in smaller market share and expected revenue for each rating agency, resulting in more ratings inflation, due to the market-sharing effect. When one rating agency is dominant and its competitor has a very low reputation, the rating agencies' market share is relatively inelastic to small changes in their reputation. Thus the disciplining effect of competition is relatively weaker and the rating agency behaves more laxly. On the other hand, if the reputation of the two rating agencies are close to each other, the rating agencies' market share is more sensitive to changes in reputation and ratings inflation is relatively smaller. On balance, our results show that the market share effect dominates the disciplining effect of competition and increasing competition results in more ratings inflation.

Mathis, McAndrews, and Rochet (2009) demonstrate that reputational concerns are not enough to solve the conflict of interest problem. In equilibrium, rating agencies are likely to behave laxly, *i.e.* rate bad projects as good and are prone to reputation cycles. Our

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<sup>4</sup>The figure stands at 95% if we include the third major player, Fitch.

<sup>5</sup>Although we only focus on competition in a duopolistic setting, our results intuitively extend to situations with higher degrees of competition.

model innovates by introducing competition through an endogenous market share function and studying how competition affects the behaviour of rating agencies.

Becker and Milbourn (2008) lends support to our results by providing an empirical test of the impact of competition on rating agencies. They measure competition using the growth of Fitch's market share and find three pieces of evidence. First, the overall standards of ratings issued by S&P and Moody's increased (closer to the top AAA rating) with competition, so that ratings are more 'friendly'. Second, the correlation between bond yields and ratings fell as competition increased, implying that ratings became less informative. Third, equity prices react more negatively to rating downgrades, suggesting a lower bar for rating categories. Their findings are consistent with our results that competition will tend to lower the quality of ratings in the market.

The adverse effects of competition on the building and maintenance of reputation has been studied by Klein and Leffler (1981). They argue that when faced with a choice between supplying high quality products or low quality ones, firms would be induced to supply high quality products when the expected value of future income given a high reputation outweighs the short-run gain of cheating. However, competition would undermine this mechanism since the expected future income would fall as competition intensifies, and hence the firm would have less incentives to maintain reputation. This is similar to our intuition that rating agencies tend to behave more laxly as competition increases. However, competition in our model also has a disciplining effect and we explore the overall impact of competition on rating agencies' behaviour.

Bolton, Freixas, and Shapiro (2009) also analyse the behaviour of strategic rating agencies in monopolistic and duopolistic settings. They look at ratings-shopping of issuers in the presence of naïve investors. They find that ratings are inflated when there are more naïve investors, and that monopoly is superior in terms of total *ex-ante* investor welfare. In addition, Skreta and Veldkamp (2008) do not consider the strategic behaviour of rating agencies but explore the interaction between ratings-shopping, complexity of the security (project) and competition. They show that the intensity for ratings-shopping increases with the complexity of the security

and that competition between rating agencies makes the problem even more acute.

Damiano, Hao, and Suen (2008) study how the rating scheme may affect the strategic behaviour of rating agencies. They compare rate inflation among centralised (all firms are rated together) and decentralised (firms are rated separately) rating schemes. When the quality of projects is weakly correlated, centralised rating dominates because decentralised rating leads to lower ratings inflation. The reverse holds when the correlation is strong.

The rest of the paper is organised as follows. In Section 2 we outline the basic features of our model. Section 3 describes the equilibrium in our model. In Section 4 we solve the model numerically in an infinite horizon. We go on to compare the behaviour of rating agencies under monopoly and duopoly in Section 4.3 and discuss the expected welfare consequences of enhanced competition. Section 5 concludes. The proofs are presented in the Appendix.

## 2 Model Setup

We consider a discrete time setting with 3 types of agents – the issuers, the rating agencies (RA) and the investors. Each period, we have a *new issuer*<sup>6</sup> with a project that requires financing. We assume that issuers do not have funds of their own and need to obtain outside financing. The investors have funds and are willing to invest in the project provided they are convinced that it is profitable to do so. The role of the RA in this setting is to issue ratings that convince investors to provide financing.

More formally, each period we have one issuer that has a project which lasts for one period. The project has a pay-off  $\Phi$  if successful and 0 otherwise and requires an investment of  $X$ , with  $X$  uniformly distributed<sup>7</sup> over  $(a,b)$ . The project is *good* with probability  $\lambda$  and *bad* with probability  $1 - \lambda$ .  $\lambda$  is independent of  $X$ . Good projects succeed with probability  $p_G$  and fail

---

<sup>6</sup>*New Issuer* implies that it is a one shot game for the issuer and we rule out the possibility that issuers try to maximise profits over multiple periods. This assumption also ensures that issuers have the same belief as the investors about the reputation of the RAs. If we allow the same issuers to approach the rating agencies in subsequent periods, then issuers will have more information than investors.

<sup>7</sup>This assumption ensures that we have a range of projects with different returns. Projects that require low investment have high return and vice versa. We can get similar results if we assume fixed investment with uncertain pay-off.

with  $1 - p_G$ . Bad projects always fail.

We assume that *a-priori* projects are not worth financing without rating, *i.e.*  $\lambda p_G \Phi \leq X$ . Further, the RAs can perfectly observe the type of project at no cost. After observing the type, the RA can either issue a good rating (GR) or no rating (NR). Note that we do not distinguish between bad rating and NR and abstract away from a ratings scale. In our setup, a good rating is one that allows the issuer to borrow from investors. It does not matter if this rating is AAA or A or BBB or even C. As long as the rating allows the firm to get financing, we consider it to be a GR. A bad rating in this setting will be a rating which does not enable a project to get financing. This is the same outcome as a NR and thus, a bad rating and NR are equivalent in our model.

The rating agency receives income  $I$  if it issues GR, and 0 otherwise.<sup>8</sup> This assumption arises from the conflict of interest in the rating agency industry. Given the *non-transparent* nature of the market and the widespread use of *negotiated ratings*, issuers and RAs routinely have negotiations and consultations before an official rating is issued. RAs, as part of their day-to-day operations, give their clients ‘creative suggestions’ on how to repackaging their portfolios/projects in order to get better ratings. To quote former chief of Moody’s, Tom McGuire<sup>9</sup>

“The banks pay only if [the rating agency] delivers the desired rating. . . . If Moody’s and a client bank don’t see eye-to-eye, the bank can either tweak the numbers or try its luck with a competitor. . . .”

We assume that there are two types of RAs - *honest* and *strategic*. An honest RA always issues a GR to a good project and NR to a bad project while a strategic RA behaves strategically to maximise its expected future profits. The strategic RA faces the following trade-off :

1. (**Truthful**) It can either be truthful and maintain its reputation, thus ensuring profits in the future

---

<sup>8</sup>This is a standard simplifying assumption in the literature. See Mathis, McAndrews, and Rochet (2009) and Skreta and Veldkamp (2008).

<sup>9</sup>New York Times Magazine, Triple-A-Failure, April 27, 2008.



2. **(Lie)** It can inflate ratings (rate a bad project good) and get fees now, at the cost of future profits

We consider a duopolistic setting with 2 rating agencies.<sup>10</sup> The type of the RA is chosen *ex ante* by nature and is known only to the rating agency itself. The *reputation* of the rating agency is defined as the probability that it is honest, denoted by  $q_i, i \in \{1, 2\}$ . The reputation evolves over time depending on the ratings and outcome of the projects. The *strategic* variable for the RA is  $x_i$ , the probability the RA issues a GR to a bad project.<sup>11</sup>

The investors (and issuers) have some priors about the types of the RAs and they Bayesian update on their beliefs. Firstly, investors and issuers take into account the rating and update the reputation of the RA, before observing the outcome of the project. Given prior reputation  $q_t$ ,

$$\text{If RA issues GR, } q_t^{GR} = \frac{\lambda q_t}{\lambda + (1 - q_t)(1 - \lambda)x} < q_t \quad (1)$$

$$\text{If not rated, } q_{t+1}^N = \frac{q_t}{1 - x(1 - q_t)} > q_t \quad (2)$$

If the project is issued a good rating by the RA, the investors update their beliefs after observing the outcome of the project.

$$\text{If the project succeeds, } q_{t+1}^S = \frac{\lambda p_G q_t}{\lambda p_G q_t + \lambda p_G (1 - q_t)} = q_t \quad (3)$$

$$\text{If the project fails, } q_{t+1}^F = \frac{\lambda(1 - p_G)q_t}{\lambda(1 - p_G)q_t + [\lambda(1 - p_G) + (1 - \lambda)x](1 - q_t)} < q_t \quad (4)$$

We make the simplifying assumption that each issuer can only approach one RA for rating. Thus we abstract away from *multiple ratings*, *herd behaviour* by RAs and *ratings-shopping* by the issuers. While these are important issues that merit attention, they are not the focus of this paper. Here we look at the competition for market share among rating agencies and show

<sup>10</sup>Given the structure of the market, with Moody's and S&P controlling nearly 80% of the market, we believe that this is a reasonable approximation of reality.

<sup>11</sup>Note that the strategic RA will always issue GR to a good project (see section 3).

that ratings inflation increases with competition.

Investors observe the rating decision and decide whether to invest. If they observe a GR from a RA with reputation  $q$ , their subjective belief that the project will succeed (using equation (1)) is given by

$$\begin{aligned}
s(q, x) &= q^{GR} p_G + (1 - q^{GR}) \frac{\lambda p_G}{\lambda + (1 - \lambda)x} \\
&= \frac{\lambda q}{\lambda + (1 - q)(1 - \lambda)x} p_G + \left(1 - \frac{\lambda q}{\lambda + (1 - q)(1 - \lambda)x}\right) \frac{\lambda p_G}{\lambda + (1 - \lambda)x} \\
&= \frac{\lambda p_G}{\lambda + (1 - q)(1 - \lambda)x}
\end{aligned} \tag{5}$$

Given the required investment level  $X$ , investors are willing to finance the project *if and only if*  $X \leq s(q, x)\Phi$ , *i.e.* if the initial investment required for the project is no greater than its expected pay-off. Without loss of generality, assume  $s(q_1, x_1) > s(q_2, x_2)$ . We have 3 cases:

1. If  $X$  is such that a good rating from either RA is enough, *i.e.*  $X \leq s(q, x)\Phi$  for both  $q_1$  and  $q_2$ , the firm can approach either RA.<sup>12</sup> We assume that in this case the firm will randomly choose one of the RAs, *i.e.* the project goes to both RAs with equal probability.<sup>13</sup>
2. If  $s(q_2, x_2)\Phi < X < s(q_1, x_1)\Phi$ , *i.e.* only the high reputation RA can issue ratings that can convince the investors to provide financing, hence the firm will go to RA1 and not RA2.
3. If  $X > s(q_1, x_1)\Phi$ , the project does not get financed.

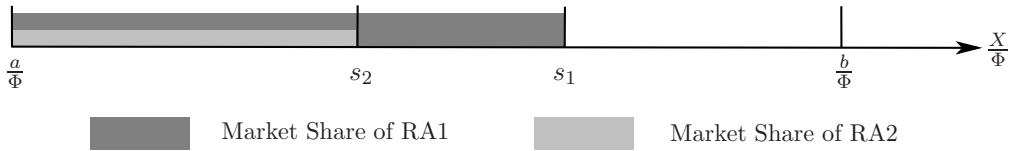


Figure 1: The Market for Ratings

<sup>12</sup>We assume that the issuers are only paid when projects succeed. This implies that the issuers will be indifferent between RAs (with different reputation) given that both can guarantee financing.

<sup>13</sup>Note that this is one of infinite many possible equilibria. Since the issuers are indifferent, we have an equilibrium for all probabilities ( $\alpha \in (0, 1)$ ) of approaching a specific RA. We focus on the case where  $\alpha = \frac{1}{2}$ . Our qualitative results do not depend on the choice of  $\alpha$ .

Thus we get the following result as illustrated in Figure 1 -

$$\begin{aligned}\text{Probability that a project comes to RA1} &= \frac{(s_1 - s_2) + \frac{1}{2}(s_2 - \frac{a}{\Phi})}{\frac{b}{\Phi} - \frac{a}{\Phi}} \\ \text{Probability that a project comes to RA2} &= \frac{\frac{1}{2}(s_2 - \frac{a}{\Phi})}{\frac{b}{\Phi} - \frac{a}{\Phi}}\end{aligned}$$

We set  $(a, b) = (\lambda p_G \Phi, p_G \Phi)$ , because any project with  $X < \lambda p_G \Phi$  does not need a rating to be financed, and any project with  $X > p_G \Phi$  is never worth financing *ex-ante*. Substituting, we get

$$\text{The probability that a project comes to RA1} = \frac{s_1 - \frac{1}{2}(s_2 + \frac{a}{\Phi})}{p_G(1 - \lambda)} \quad (6)$$

$$\text{The probability that a project comes to RA2} = \frac{\frac{1}{2}(s_2 - \frac{a}{\Phi})}{p_G(1 - \lambda)} \quad (7)$$

Reputation plays a critical role in our model. The market share of the RAs depends on  $s$ , and thus on reputation  $q$ . Since the income from giving a GR is constant (denoted by  $I$ ), the future profits of the RA will solely depend on its market share. Moreover, the RA with a higher reputation enjoys additional benefits of being the market leader, because it owns entirely the proportion of the market that cannot be possibly rated by its competitor but can be rated by itself, whereas its competitor can only share its market with the leader. This creates incentives for RAs to maintain or gain the market leader position and hence disciplines the RAs through competition.

We can now see that competition (modelled through market share) has two effects on lax behaviour: the market-sharing effect and the disciplining effect. The market-sharing effect refers to the fact that the RA finds lying and receiving income today more attractive as its expected future income is shared with another RA, and the disciplining effect refers to the fact that the RA finds lying less attractive in order to maintain/gain the advantages of being a market leader. We will show later that the market-sharing effect tends to dominate the disciplining effect and hence competition aggravates the lax behaviour of RAs in general.

### 3 Equilibrium Definition

**Definition 1.** *The equilibrium in our model is a set of strategies such that: At each period  $t$ , the strategic RA always*

(i) *Gives a good rating to a good project.*

(ii) *Gives a good rating to a bad project with probability  $x_t$ , where  $0 \leq x_t \leq 1$ .*

*The strategic RA does not have any incentives to deviate from the above strategy.*

Let RA1 be a strategic RA and let  $V_t(q_1, q_2)$  denote its discounted future profits, given its reputation  $q_1$  and its competitor's reputation  $q_2$ , and let  $\delta$  be the discount rate. The RA's new reputation after it gives NR and the failure of a project following a GR are denoted by  $q_1^N$  and  $q_1^F$  respectively.<sup>14</sup> Note that  $q_1^F$  and  $q_1^N$  are functions of the strategy of the RA and its current reputation level. For notational simplicity, we suppress the time subscript of these reputation-updating functions.

Figure 2 shows the decision tree of RA1. Suppose it is approached for rating. If the project is good, RA1 gives it a GR and gets income  $I$ .<sup>15</sup> On the other hand, if the project is bad, RA1 strategically decides whether to give a GR and get fees  $I$  or refuse rating. In case of NR, RA1's reputation rises as it gets a larger market share in the future. In case of a GR, RA1's reputation falls if the project fails and remains the same if it succeeds. This in turn determines the RA1's expected future income. A similar analysis applies if RA2 is approached for rating. In this case the fees go to RA2 and RA1 is only indirectly affected through a change in RA2's reputation. Note that since RA1 does not know the type of RA2, it has to take into account the possibility that RA2 is either honest or strategic.

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<sup>14</sup>A successful project with a GR leaves the RA's reputation unchanged.

<sup>15</sup>see Proposition 2

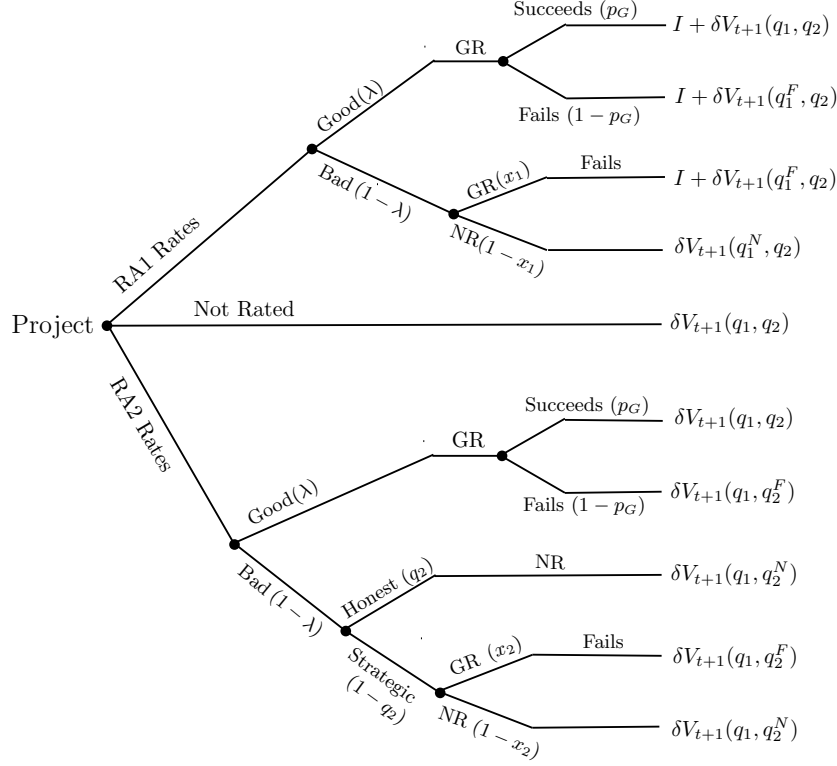


Figure 2: Decision Tree for strategic RA1

$$\begin{aligned}
V_t(q_1, q_2) = & P(RA1rates) \left\{ P(Good) [I + p_G \delta V_{t+1}(q_1, q_2) + (1 - p_G) \delta V_{t+1}(q_1^F, q_2)] \right. \\
& + P(Bad) [x_1(q_1, q_2) (I + \delta V_{t+1}(q_1^F, q_2)) + (1 - x_1(q_1, q_2)) \delta V_{t+1}(q_1^N, q_2)] \left. \right\} \\
& + P(RA2rates) \left\{ P(Good) [p_G \delta V_{t+1}(q_1, q_2) + (1 - p_G) \delta V_{t+1}(q_1, q_2^F)] \right. \\
& + P(Bad) [(1 - q_2) x_2(q_1, q_2) \delta V(q_1, q_2^F) + [q_2 + (1 - q_2) (1 - x_2(q_1, q_2))] \delta V(q_1, q_2^N)] \left. \right\} \\
& + P(NotRated) \delta V_{t+1}(q_1, q_2) \quad (8)
\end{aligned}$$

The objective function of RA1 is to maximise  $V_t(q_1, q_2)$ , the choice variable being  $x_1$ . Note that RA1's choice variable is only effectual when it rates a bad project. In all other cases,

RA1's strategy is inconsequential.

**Proposition 1.** *There exists a unique  $x_1$ , where  $0 \leq x_1 \leq 1$ , given that  $V_t(q_1, q_2)$  is an increasing function in  $q_1$ .*

*Proof.* See Appendix A.1 □

Intuitively, it is easy to see from equation (8) that  $V_t(q_1, q_2)$  is linear in  $x_1$ . This ensures that RA1's maximisation problem has a unique solution.

**Proposition 2.** *A strategic RA does not have incentives to give NR to a good project.*

*Proof.* See Appendix A.2 □

Proposition 2 implies that a strategic RA always gives GR to a good project. This is because it gets a lower pay-off if it deviates from this strategy and gives a NR to a good project. The proposition follows directly from the pay-off structure of the RAs and the beliefs.

**Proposition 3.** *There exists a unique equilibrium as described in Definition 1.*

*Proof.* Follows from propositions 1 and 2. □

**Corollary 1.** *Assume  $p_G < 1$ . Then the equilibrium strategy of the strategic RA is always positive.*

*Proof.* See Appendix A.3 □

**Corollary 2.** *Suppose the model ends in period  $T$ . Then the equilibrium strategy of the strategic RA is  $x = 1$  at  $t = T - 1, T$ .*

*Proof.* See Appendix A.4 □

We now solve the model numerically in infinite horizon. We present an analytical solution in a finite period setting in Appendix A.5.

## 4 Model Solution

We now present the numerical solution of the model in infinite horizon. The numerical solution is computed using backward induction, *i.e.* we first solve the model in the finite period case, and then increase the number of periods so that the equilibrium strategy converges to the infinite horizon solution.

In an infinite period setting,  $V_t$  by itself is independent of  $t$ . Hence we suppress the time subscript for notational simplicity. However, the reputations evolve over time as investors (and issuers) update their beliefs. Let RA1 be the rating agency that behaves strategically. Then, RA1's value function takes the following form:

$$\begin{aligned}
 V(q_1, q_2) = & \frac{\frac{1}{2}(s_1 - \frac{a}{\Phi})}{(1-\lambda)p_G} \left\{ \lambda \left[ I + p_G \delta V(q_1, q_2) + (1 - p_G) \delta V(q_1^F, q_2) \right] + \right. \\
 & (1 - \lambda) \left[ x_1(q_1, q_2) (I + \delta V(q_1^F, q_2)) + (1 - x_1(q_1, q_2)) \delta V(q_1^N, q_2) \right] \left. \right\} \\
 & + \frac{s_2 - \frac{1}{2}(s_1 + \frac{a}{\Phi})}{(1-\lambda)p_G} \left\{ \lambda \left[ p_G \delta V(q_1, q_2) + (1 - p_G) \delta V(q_1, q_2^F) \right] + \right. \\
 & (1 - \lambda) \left[ (1 - q_2) x_2(q_1, q_2) \delta V(q_1, q_2^F) + [q_2 + (1 - q_2)(1 - x_2(q_1, q_2))] \delta V(q_1, q_2^N) \right] \left. \right\} \\
 & + \frac{p_G - s_2}{(1-\lambda)p_G} \delta V(q_1, q_2) \quad (9)
 \end{aligned}$$

where  $\frac{\frac{1}{2}(s_1 - \frac{a}{\Phi})}{(1-\lambda)p_G}$  is the probability that the issuer approaches RA1 for rating,  $\frac{s_2 - \frac{1}{2}(s_1 + \frac{a}{\Phi})}{(1-\lambda)p_G}$  is the probability that the issuer approaches RA2 and  $\frac{p_G - s_2}{(1-\lambda)p_G}$  is the probability that the project is not rated by either RA.

We assume that the model ends at period  $T$  and solve the model backwards. We know that the strategic RA will always lie at period  $T$  and  $T - 1$  according to Corollary 2. For all  $t < T - 1$ , the strategy of the RA depends on its own and its competitors' reputation. We solve for the Nash equilibrium strategy of the RA described in Section 3. We look at the pay-offs from lying and being honest and determine the strategy. As long as  $I + V_t(q_1^F, q_2) > V_t(q_1^N, q_2)$

for  $x_t = 1$ , RA1 will always choose to lie. Conversely, if  $I + V_t(q_1^F, q_2) < V_t(q_1^N, q_2)$  for  $x_t = 0$ , RA1 will always tell the truth. In all other intermediate cases, there exists a unique  $x_t$  *s.t.*  $I + V_t(q_1^F, q_2) = V_t(q_1^N, q_2)$  at which RA1 is indifferent between lying or not. Hence we deduce inductively the equilibrium strategies of RA1. As  $T$  goes to infinity, we approach the infinite horizon solution.<sup>16</sup>

Using this procedure, we solve the model for various parameter values. At the first instance, we solve the model for a monopolistic RA. Next, we introduce competition in the form of RA2 and show that the additional competitive element is not sufficient to discipline the RAs. Furthermore, our results show that competition will in fact increase ratings inflation.

#### 4.1 Monopolistic RA

First we consider the case where there is only one RA in the market. In order to make RA1 a monopolist, we set the reputation of RA2 to 0.

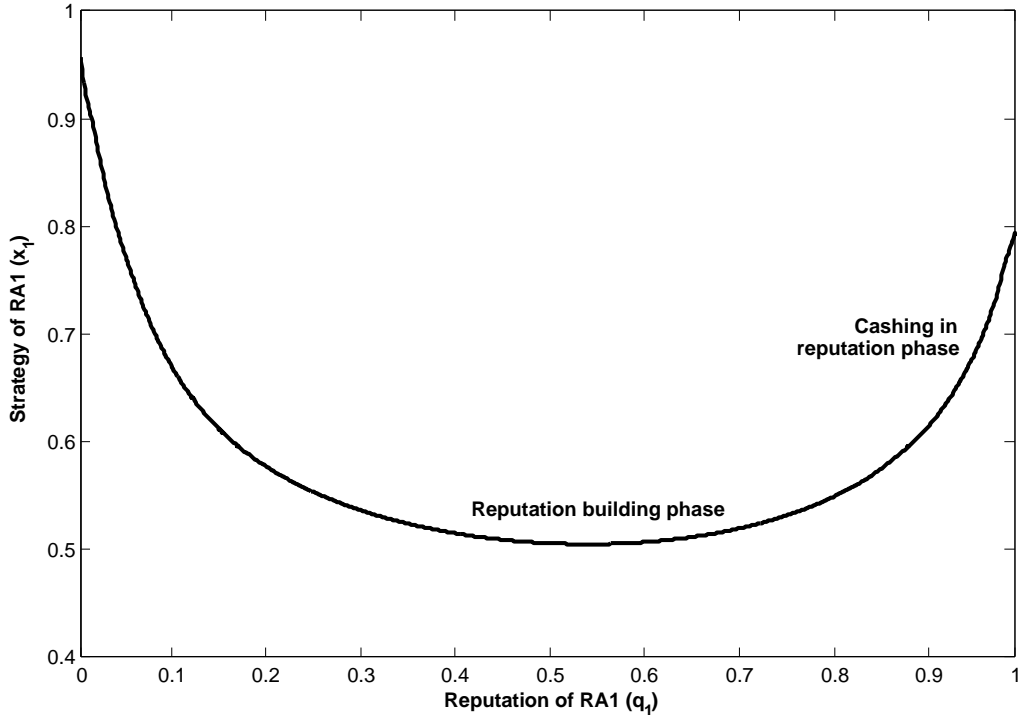


Figure 3: Strategy vs Reputation, Monopolistic RA  $(\lambda, p_G, \delta, q_2) = (0.5, 0.7, 0.9, 0)$

<sup>16</sup>Since  $\delta < 1$ , the Blackwell conditions are satisfied.



Figure 3 plots the strategy of the monopolistic RA for parameters  $(\lambda, p_G, \delta) = (0.5, 0.7, 0.9)$ .<sup>17</sup> We can clearly see the strategy of RA1 is ‘u-shaped’ in its reputation and it tends to lie more when its reputation is very high or very low. Intuitively, the RA’s strategy is determined by the trade-off between current fees and expected future income. When its reputation is very low, the RA’s expected future income is very small compared to current fees, hence it has little incentive to behave honestly. When its reputation increases, the RA’s future income becomes larger while current fees stay the same, the RA tends to lie less. However, when the RA’s reputation becomes very high, the penalty for lying decreases, therefore the RA starts to lie more. The reason that the penalty for lying decreases with reputation is that investors attribute project failures to bad luck rather than lax behaviour when they believe that the RA is very likely to be of the honest type.

## 4.2 Competitive RA

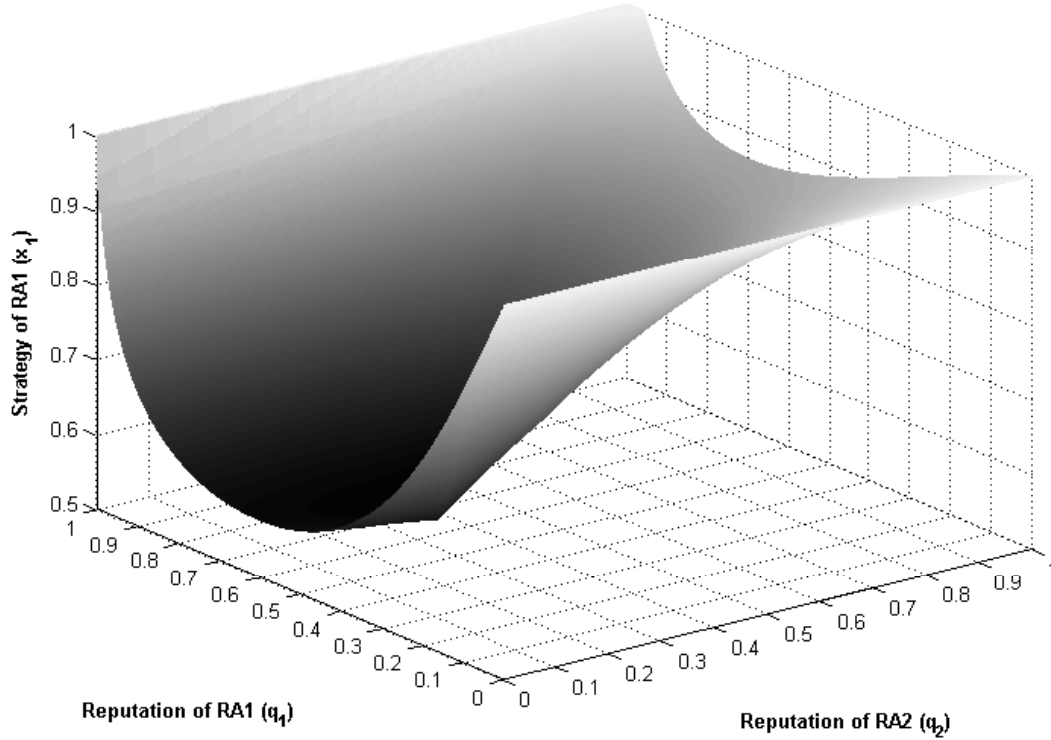


Figure 4: Strategy vs Reputation,  $(\lambda, p_G, \delta) = (0.5, 0.7, 0.9)$

<sup>17</sup>Note that our qualitative results are robust to parameter specification.

We now look at the impact of competition on the behaviour of rating agencies by introducing a second RA (RA2). Figure 4 plots the strategy of RA1 for parameter values  $(\lambda, p_G, \delta) = (0.5, 0.7, 0.9)$ . Figures 5 and 6 show cross-sections of this figure, for different values of  $q_2$  and  $q_1$  respectively.

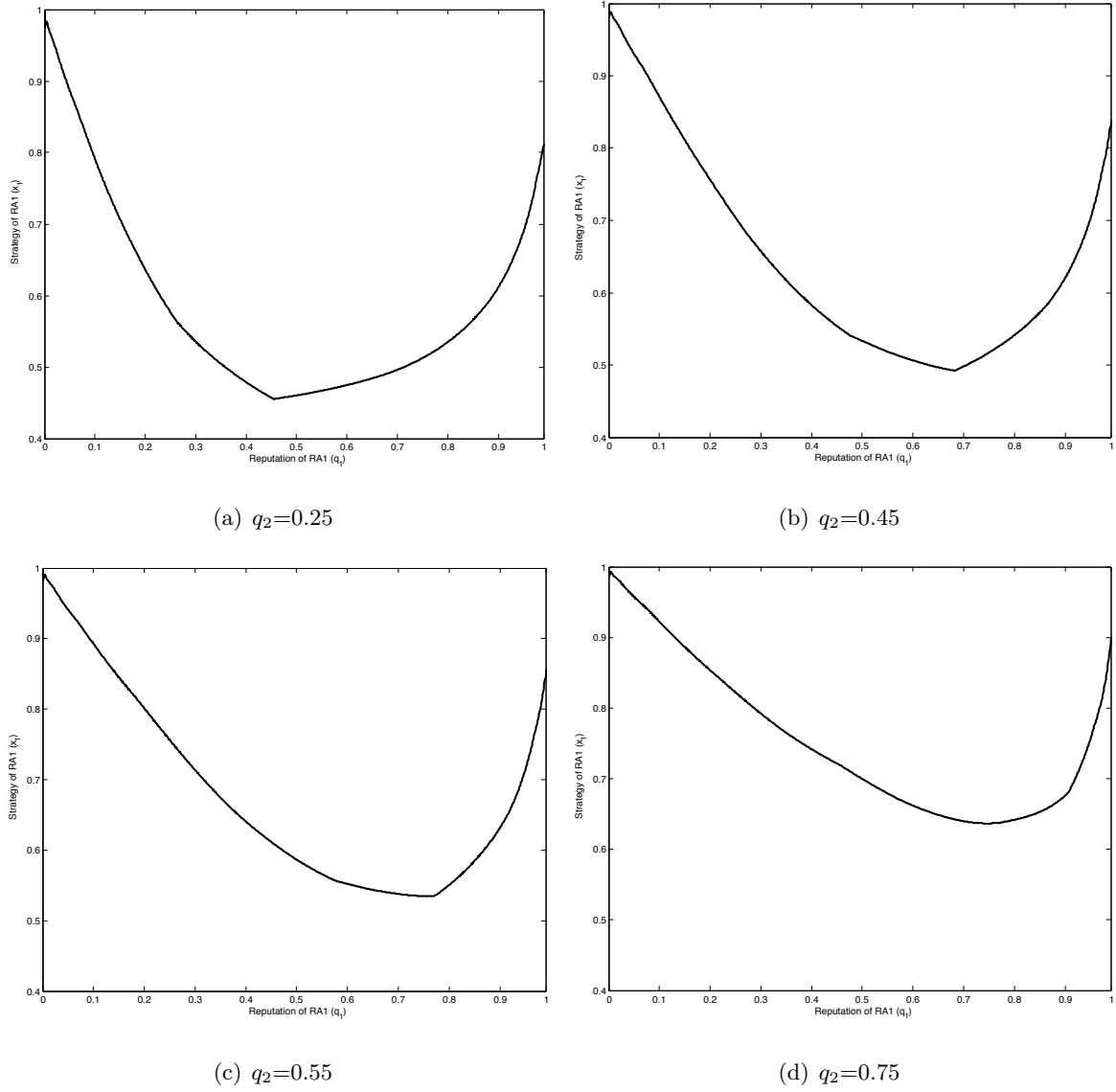


Figure 5: Strategy vs Reputation,  $(\lambda, p_G, \delta) = (0.5, 0.7, 0.9)$ , different values of  $q_2$

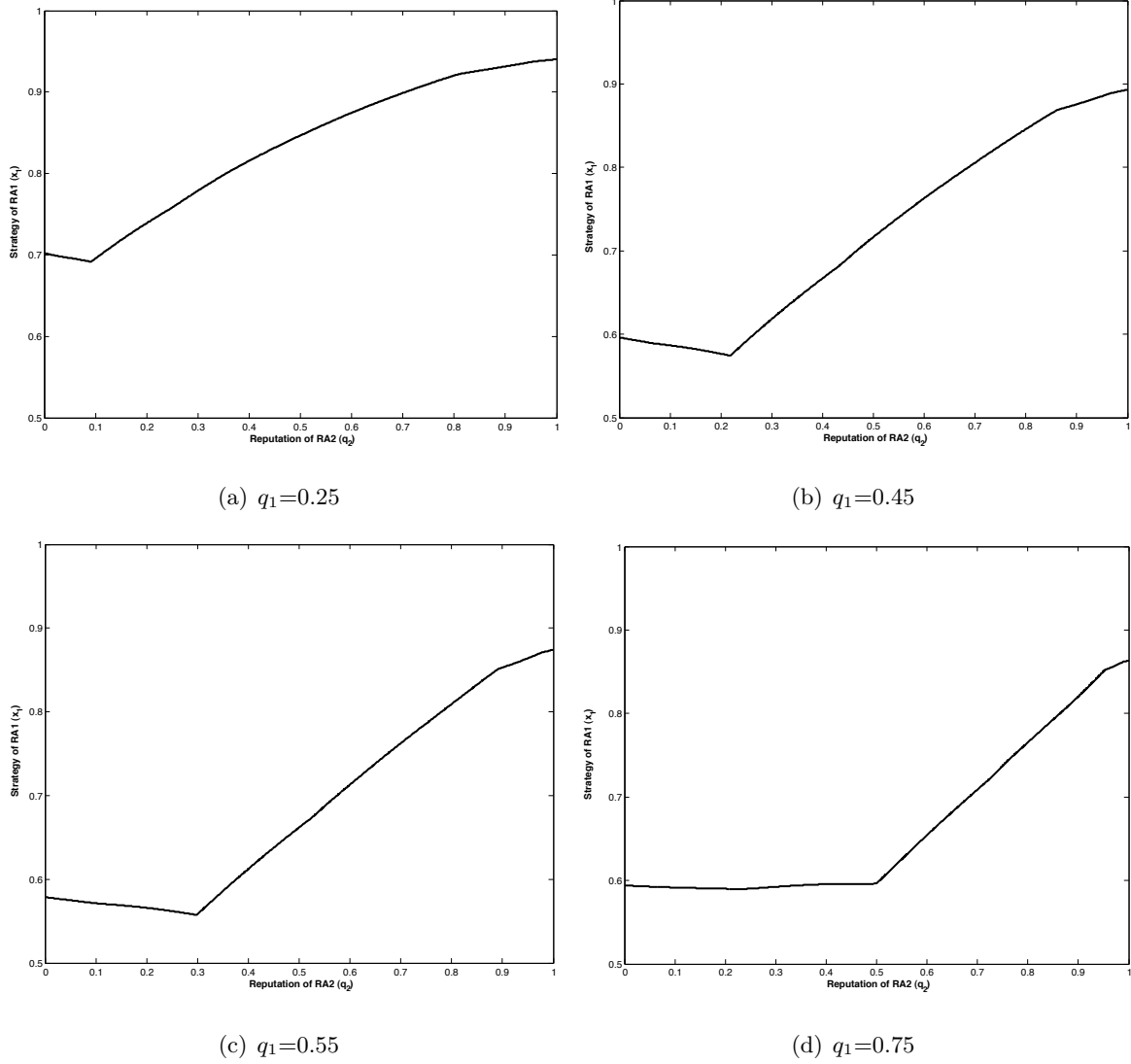


Figure 6: Strategy vs Reputation,  $(\lambda, p_G, \delta) = (0.5, 0.7, 0.9)$ , different values of  $q_1$

Figure 5 shows the relationship between the reputation and strategy of RA1 for different values of the competing RA2's reputation. As we can see, the relationship between the reputation and strategy of RA1 remains 'u-shaped' as in the monopolistic case. Moreover, as the reputation of RA2 increases, the reputation at which RA1 has minimum  $x_1$ , *i.e.* is least likely to lie, also increases. This is not surprising as the disciplining effect is greatest when the reputation of the competing RA (RA2) is close to the reputation of RA1. This is because when the RAs' reputations are close, it is more likely that the market leadership will change, resulting

in more disciplined behaviour. Conversely, if the two RAs have very different reputations, the disciplining effect is relatively weaker.

Moreover, as Figure 6 shows, the strategy of RA1 is initially decreasing with or flat in RA2's reputation, and then increasing. This effect of competition is a combination of the disciplining effect and the market-sharing effect. The disciplining effect is strongest when the two RA's reputations are close, and weakest when the two RA's reputations are far apart, which implies that the probability of a change of market leader is very small. On the other hand, the market-sharing effect is always increasing in the competing RA's reputation. When the reputation of RA2 is low, the market-sharing effect is very small as RA2 can only take away a tiny fraction of market share. As RA2's reputation starts to increase, RA1 tends to lie less as the disciplining effect dominates the market-sharing effect. However, when RA2's reputation goes beyond a certain level, the market-sharing effect dominates as RA2's reputation becomes much higher than RA1's. Hence RA1 will lie more for high values of RA2's reputation, due to the dominance of the market-sharing effect.

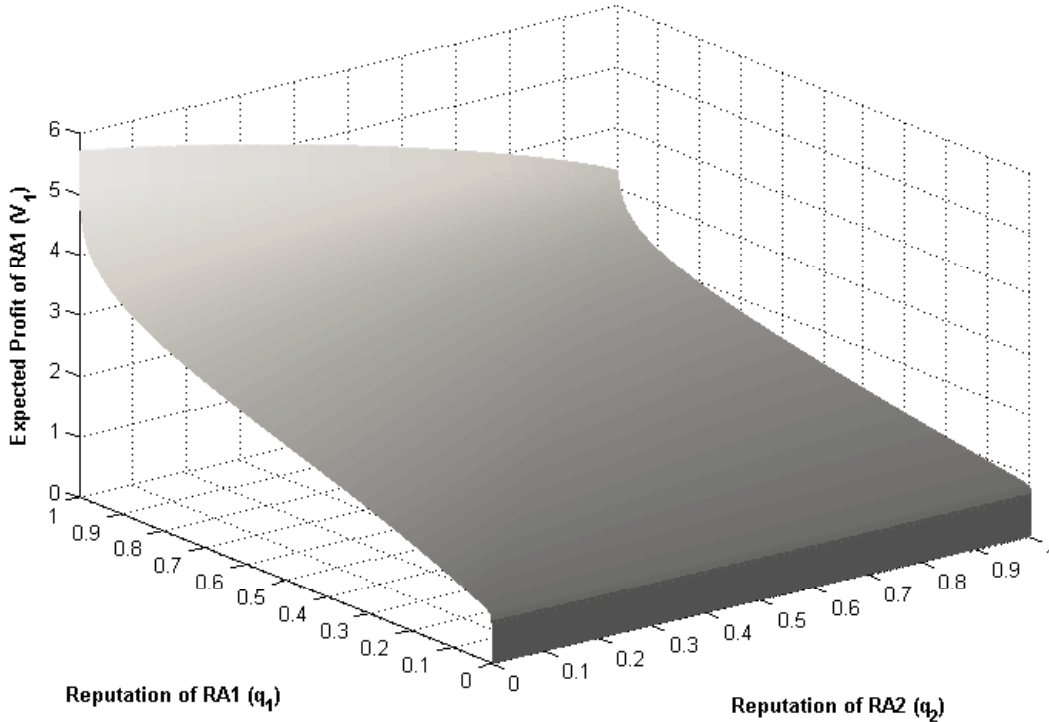


Figure 7: Expected Profits vs Reputation,  $(\lambda, p_G, \delta) = (0.5, 0.7, 0.9)$

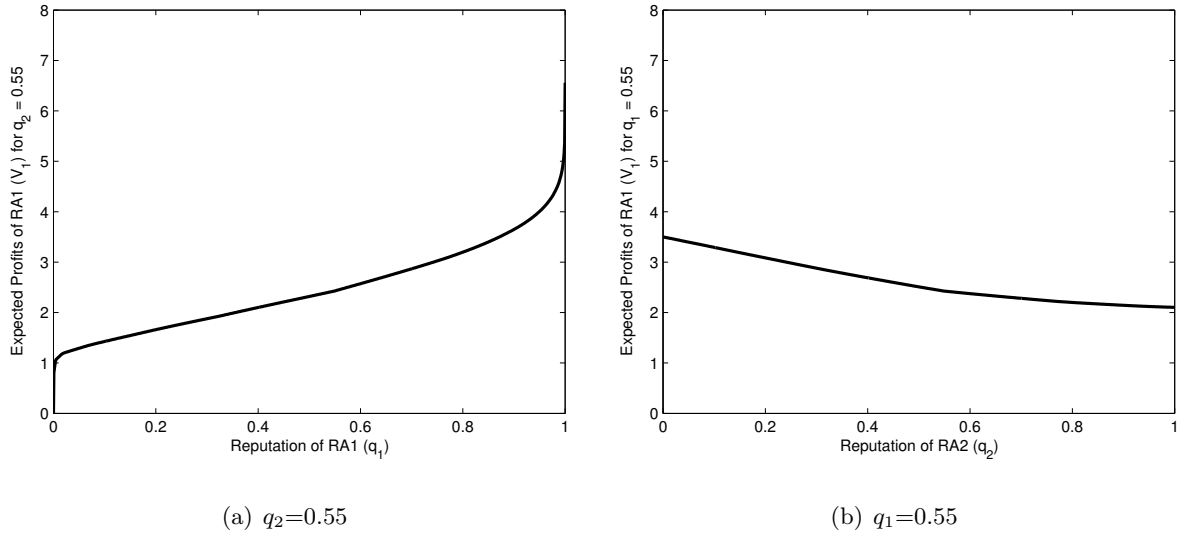


Figure 8: Expected Profits vs  $q_1$  and  $q_2$ ,  $(\lambda, p_G, \delta) = (0.5, 0.7, 0.9)$

Figures 7 and 8 show the expected profits of RA1 as a function of RA1 and RA2's reputation. We can clearly see that the expected profits of RA1 is increasing in its own reputation, and decreasing in its competitor's reputation, illustrating the market-sharing effect.

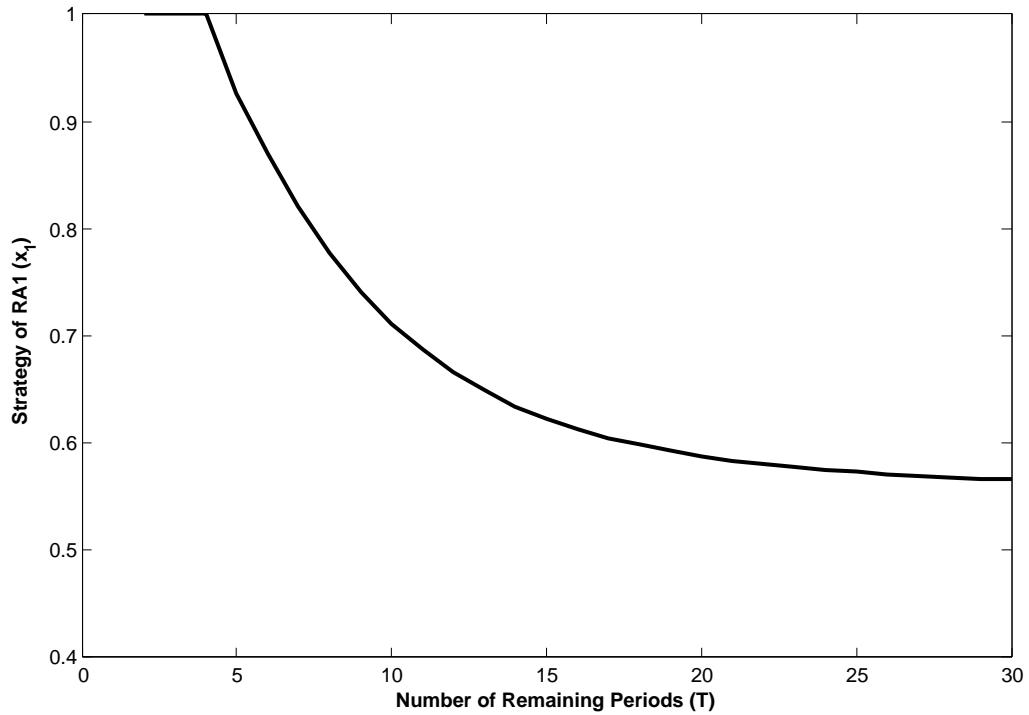


Figure 9: Convergence Dynamics of RA1

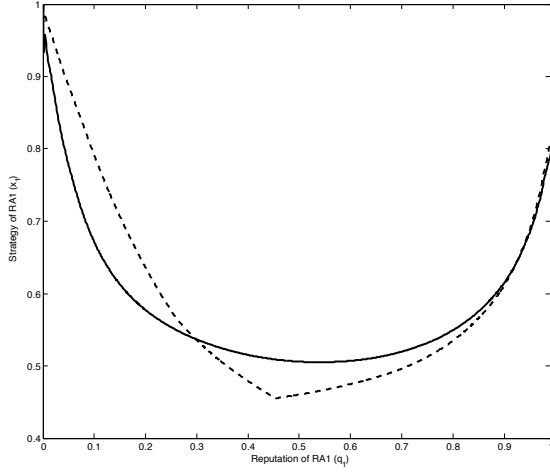
Finally, Figure 9 shows the convergence dynamics. It plots the change in RA1's strategy as the number of periods remaining increases. Reputation becomes less and less important as the number of periods remaining declines since there are fewer periods to reap the benefits of higher reputation. Thus ratings inflation increases. Note that as the number of periods remaining increases, the strategy converges, implying that we approach a long (infinite) horizon equilibrium.

In summary, our results show that introducing competition in the form of a second RA is not sufficient to discipline the RAs which always lie with positive probability in equilibrium. We now show that competition will actually increase the lax behaviour of RAs and reduce expected welfare.

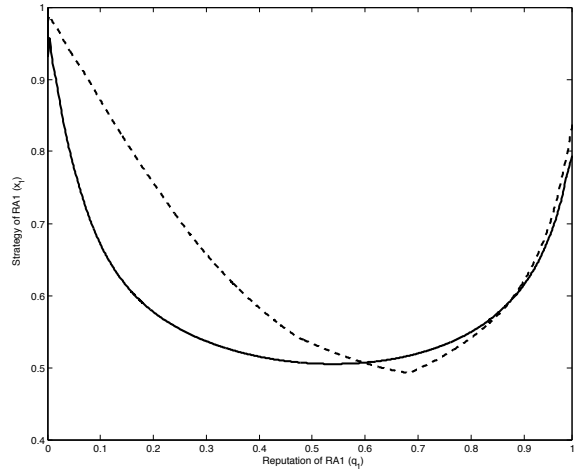
### 4.3 Comparing Monopolistic and Competitive RA

It is often suggested that introducing more competition in the ratings industry can alleviate the problem of improper incentives and ratings inflation. However, our results show that competition is likely to worsen this situation and lead to more ratings inflation.

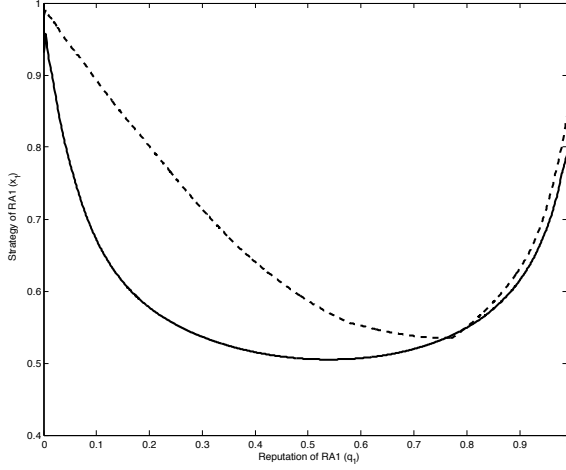
Figure 10 compares the strategic behaviour of RA1 under no competition, *i.e.* monopolistic RA ( $q_2 = 0$ ), and under a competitive setting with different values of  $q_2$ . We observe that in most cases, RA1 is prone to greater ratings inflation relative to the monopolistic RA.



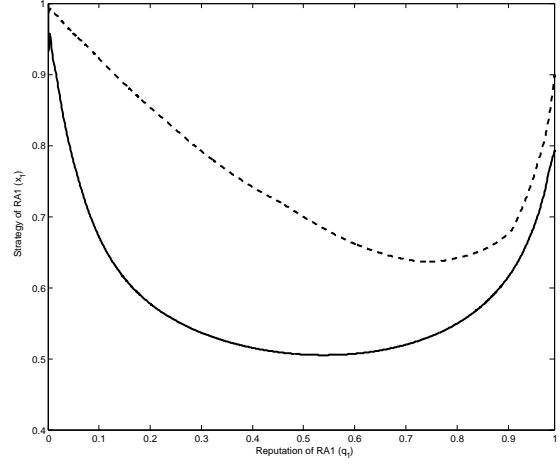
(a)  $q_2=0.25$



(b)  $q_2=0.45$



(c)  $q_2=0.55$



(d)  $q_2=0.75$

Figure 10: Comparing Monopolistic and Competitive RA,  $(\lambda, p_G, \delta) = (0.5, 0.7, 0.9)$

Solid line represents monopolistic RA while dashed line represents competitive RA with different values of  $q_2$

As described before, the implication of competition can be divided into the market-sharing effect and the disciplining effect. We can see that the market-sharing effect dominates the disciplining effect (*i.e.* competition aggravates lax behaviour) in most cases. The only case where competition may actually alleviate the lax behaviour of RA1 is when  $q_2$  is very low (as shown in Figure 10(a)). This is because the market-sharing effect is weakest relative to the disciplining effect for low values of  $q_2$ . Intuitively, the disciplining effect only depends on the

difference between  $q_1$  and  $q_2$ , whereas the market-sharing effect increases with the absolute level of  $q_2$ . Hence the market-sharing effect tends to dominate the disciplining effect except for low values of  $q_2$ .

In order to assess the overall impact of competition, we compute the expected increase in lax behaviour of RA1 given its own reputation, assuming that the reputation of RA2 is uniformly distributed on  $[0, 1]$ . A positive value of this measure means the overall effect of enhanced competition on RA1 is to lie more (*i.e* inflate ratings more).

$$\text{Excess Lax Behaviour of RA1} = \int_{q_2 \in [0,1]} x_1(q_1, q_2) dq_2 - x_1(q_1, 0) \quad (10)$$

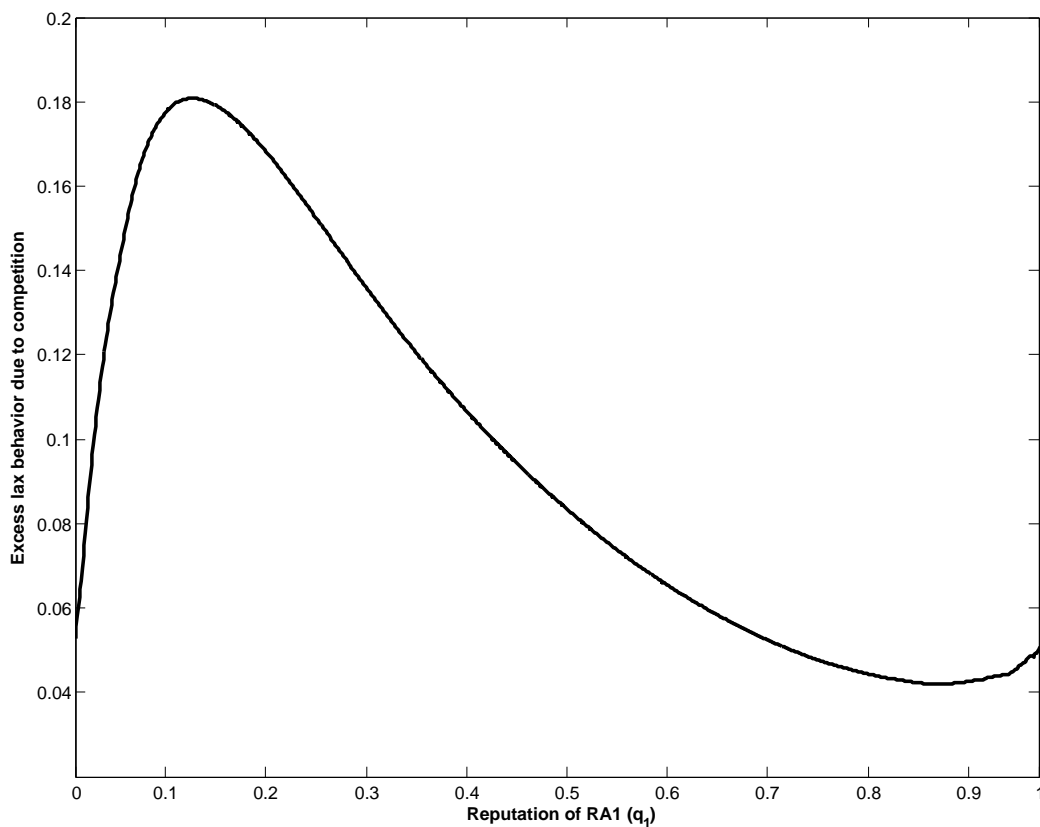


Figure 11: Excess Lax Behaviour of RA1 Due to Competition

As shown in Figure 11, the expected increase in lax behaviour of RA1 is always positive,



indicating that competition will, in general, aggravate ratings inflation. This is because a smaller market share will tend to reduce the reputational concerns of the RAs, and this market-sharing effect outweighs the disciplining effect brought by competition. Moreover, we can see that the expected increase in lax behaviour is increasing for low values of RA1's own reputation and decreasing for high values of RA1's reputation. The intuition is that, when the reputation of RA1 is low, the market share of RA1 is going to shrink significantly after introducing RA2 and the market-sharing effect of competition is strongest. However, when the reputation of RA1 is high, the impact of introducing RA2 on RA1's market share is small, hence the market-sharing effect becomes weaker and RA1 will lie relatively less.

In addition, we measure the expected total welfare in the monopolistic and duopolistic settings as defined below.

$$\begin{aligned} \text{Expected Total Welfare} &= E(\text{Project Payoff}) - E(\text{Financing Cost}) \\ &= P(\text{RA1 rates}) \left( P(\text{Good})\Phi - E(X)(P(\text{Good}) + P(\text{Bad})(1 - q_1)x_1) \right) \\ &\quad + P(\text{RA2 rates}) \left( P(\text{Good})\Phi - E(X)(P(\text{Good}) + P(\text{Bad})(1 - q_2)x_2) \right) \end{aligned}$$

Figure 12 compares the total welfare<sup>18</sup> between the monopolistic case and the duopolistic case where both RAs have the same reputation. We can see that if a new RA is introduced with the same reputation as the incumbent RA, then the total welfare will always decrease, due to the fact that both RAs are more likely to inflate ratings.

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<sup>18</sup>We are computing the welfare in one period only because it does not depend on time.

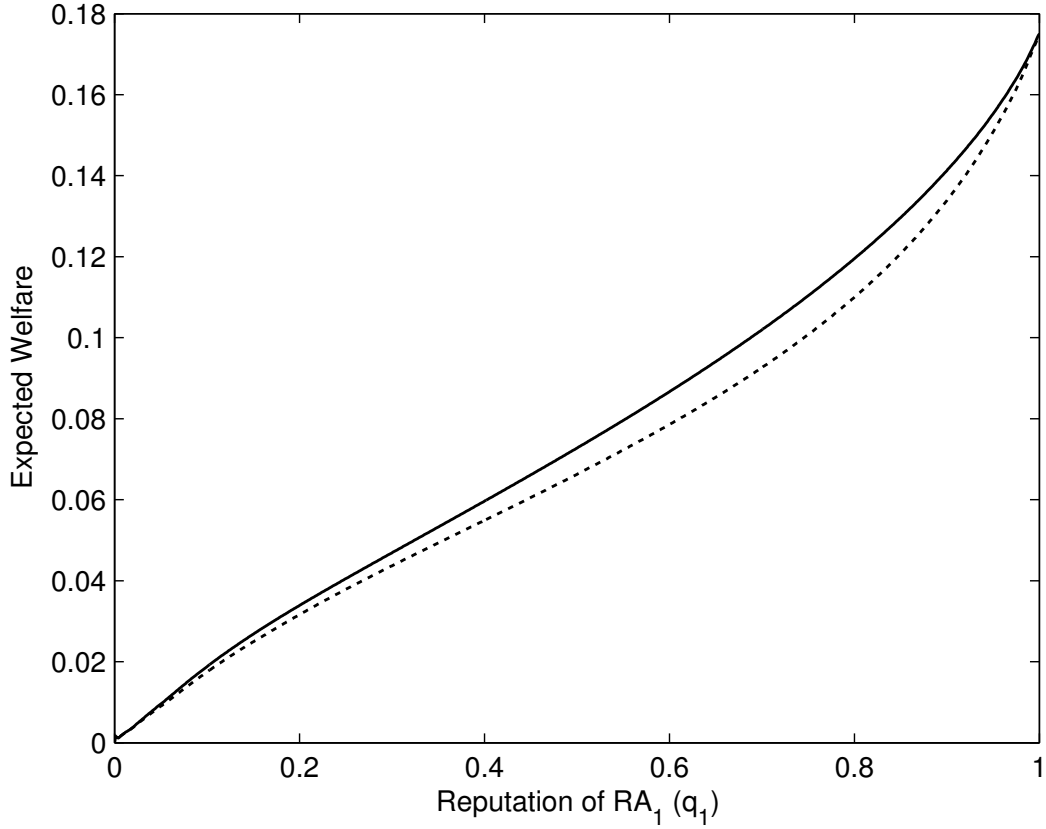


Figure 12: Expected Welfare - Competitor has same Reputation  
Solid line represents monopoly while dashed line represents duopoly with  $q_1 = q_2$

Moreover, we compare in Figure 13 the expected total welfare between the monopolistic case and the duopolistic case with fixed values of reputations of RA2. We can see that introducing competition will always lead to lower total welfare as long as the reputation of RA2 is lower than the reputation of RA1.<sup>19</sup> More importantly, even when RA2 has a higher reputation than RA1, the total welfare may still decrease in some cases. This implies that competition may adversely impact total welfare even when we introduce a new RA with higher reputation.

<sup>19</sup>The reason that total welfare may increase for very low values of  $q_1$  is that we are introducing a new RA that is much more likely to be honest.

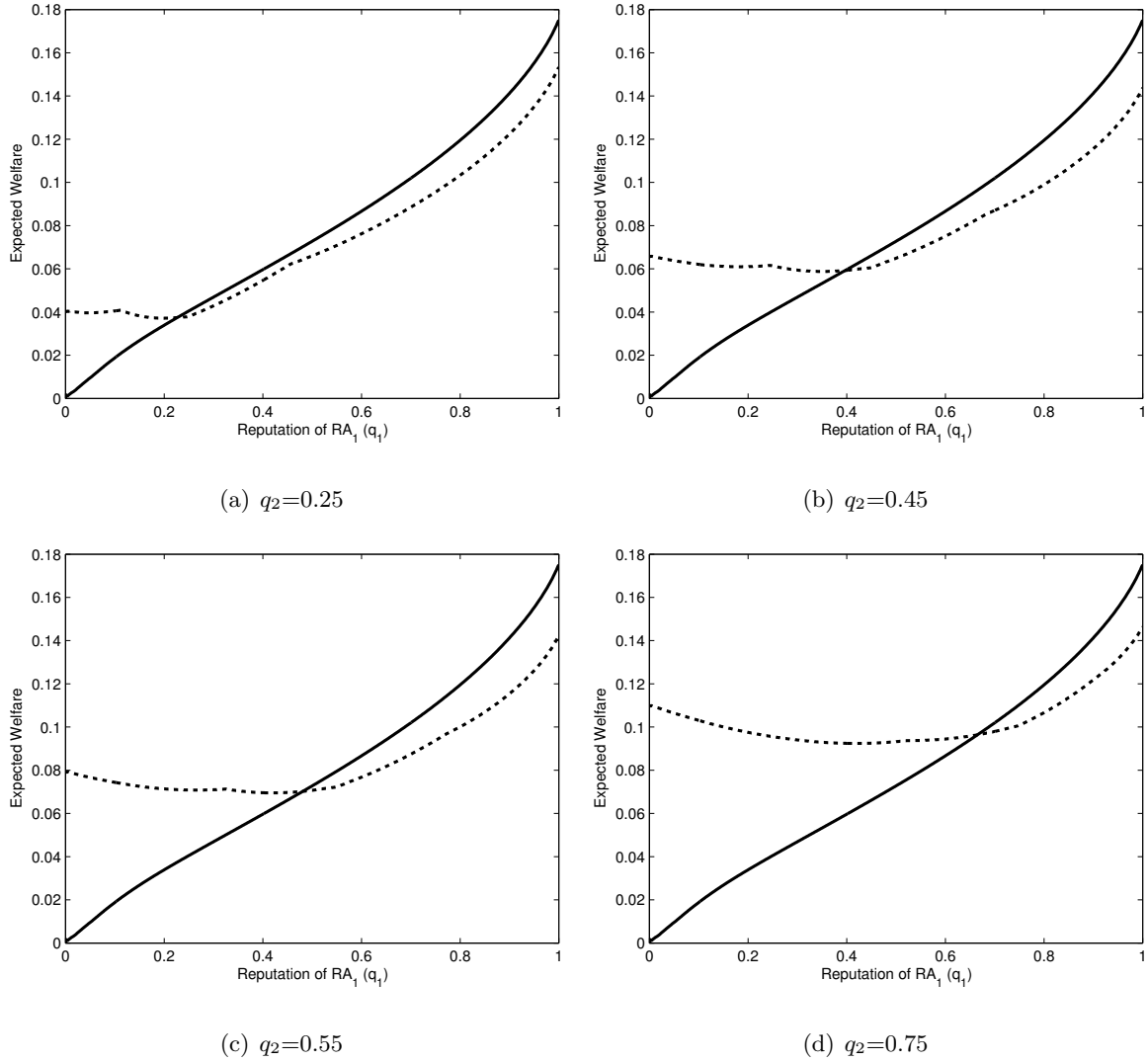


Figure 13: Expected Welfare - Competitor has Different Reputation  
Solid line represents monopoly while dashed line represents duopoly for different values of  $q_2$

## 5 Conclusion

In this paper we show that competition tends to amplify the lax behaviour of rating agencies and reduce total welfare. This result has important policy implications since it suggests that the most often cited solution to ratings inflation - enhanced competition in the ratings industry - is likely to render the situation worse.

While we acknowledge that in order to focus on the implications of competition in the rating agency industry, we have abstracted from other important issues such as herd behaviour, ratings-shopping and the quality of the models used by rating agencies, we hope that our results can serve as a baseline for evaluating the reform proposals currently being discussed. In conjunction with related work on rating shopping and herd behaviour in the rating industry, our results suggest that a fundamental reorganisation of the ratings industry may be required to align the incentives. The conflict of interest highlighted in our paper is fundamental to the issuer-pay model and any meaningful attempt to resolve the conflict would require a fundamental shift in the way rating agencies are compensated.<sup>20</sup>

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<sup>20</sup>See Deb and Murphy (2009) for a detailed study of the policy implications of our results and a proposal to reorganise the ratings industry.

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## A Appendix

### A.1 Proof of Proposition 1

There exists a unique  $x_1$ , where  $0 \leq x_1 \leq 1$ , given that  $V_t(q_1, q_2)$  is an increasing function in  $q_1$ .

*Proof.* When the strategic RA (RA1) gets a bad project, it will get pay-off  $\Psi(lie) = I + \delta V_t(q_1^F, q_2)$  if it gives the project a GR, and  $\Psi(honest) = \delta V_t(q_1^N, q_2)$  if it refuses rating. Note that  $q_1^F = \frac{\lambda(1-p_G)q_t}{\lambda(1-p_G)+(1-\lambda)(1-q_1)x_1}$  and  $q_1^N = \frac{q_t}{1-x_1(1-q_t)}$ , i.e.  $q_1^F$  is decreasing in  $x_1$  and  $q_1^N$  is increasing in  $x_1$ . Given that  $V_t(q_1, q_2)$  is increasing in  $q_1$ , it is easy to see that  $\Psi(lie)$  is decreasing in  $x_1$  and that  $\Psi(honest)$  is increasing in  $x_1$ . Thus if we define  $x_1$  such that

- $x_1 = 1$  if  $\Psi(lie) \geq \Psi(honest)$
- $x_1 = 0$  if  $\Psi(lie) \leq \Psi(honest)$  for
- $x_1 = x_1^*$  such that  $0 < x_1^* < 1$  if  $\Psi(lie) = \Psi(honest)$

it follows that  $x_1$  is well-defined and unique. □

### A.2 Proof of Proposition 2

The strategic RA does not have incentives to give NR to a good project.

*Proof.* Suppose that the strategic RA (RA1) gets a good project and that its strategy is  $x_1$ .

Let's examine whether RA1 wants to deviate:

- if  $x_1 = 1$ , we have  $\Psi(lie) \geq \Psi(honest)$ , or  $I + \delta V_t(q_1^F, q_2) \geq \delta V_t(q_1^N, q_2)$ . If the RA1 gives NR to the good project, it will get  $\delta V_t(q_1^N, q_2)$ , and  $I + p_G \delta V_t(q_1, q_2) + (1 - p_G) \delta V_t(q_1^F, q_2)$  otherwise. Since  $I + p_G \delta V_t(q_1, q_2) + (1 - p_G) \delta V_t(q_1^F, q_2) \geq I + \delta V_t(q_1^F, q_2) \geq \delta V_t(q_1^N, q_2)$ , RA1 does not want to deviate.
- if  $x_1 = 0$ ,  $q_1^N = q_1^F = q_1$ , hence reputation becomes irrelevant and the RA does not have an incentive to give NR to the good project.
- if  $0 < x_1 < 1$ , we have  $\Psi(lie) = \Psi(honest)$ , so  $I + p_G \delta V_t(q_1, q_2) + (1 - p_G) \delta V_t(q_1^F, q_2) \geq I + \delta V_t(q_1^F, q_2) = \delta V_t(q_1^N, q_2)$ , and hence RA1 does not want to deviate.

Therefore RA1 does not have incentives to give NR to a good project.  $\square$

### A.3 Proof of Corollary 1

Assume  $p_G < 1$ . Then the equilibrium strategy of the strategic RA is always positive.

*Proof.* Suppose that the equilibrium strategy is  $x_1 = 0$ . Then  $q_1^N = q_1^F = q_1$  and we must have  $I + \delta V_t(q_1, q_2) \leq \delta V_t(q_1, q_2)$ . This is impossible as long as  $I > 0$ . Hence  $x_1 = 0$  cannot be an equilibrium strategy.  $\square$

### A.4 Proof of Corollary 2

Suppose the model ends in period  $T$ . Then the equilibrium strategy of the strategic RA is  $x_t = 1$  at  $t = T - 1, T$ .

*Proof.* At  $t = T$ , the strategic RA does not have any reputational concerns. This implies that the strategy of strategic RA will be to always give GR if the project is bad, *i.e.*  $x_T = 1$ .



Similarly, at  $t = T - 1$  the strategic RA will always lie. Suppose that a bad project comes to strategic RA, say RA1. The expected pay-off of RA1 is

$$I + \delta V_{T-1}(q_1^F, q_2) = I + f(q_1^F, 1, q_2, 1)\delta I \quad (11)$$

if it lies, *i.e.* gives a good rating, and

$$\delta V_{T-1}(q_1^N, q_2) = f(q_1^N, 1, q_2, 1)\delta I \quad (12)$$

if it does not lie, *i.e.* gives no rating, where  $f(q_1, x_1, q_2, x_2)$  is the probability that the project comes to RA1 in the next period. Using equations (5), (6) and (7) we have

- $f(q_1, x_1, q_2, x_2) = \frac{\frac{1}{2}(s(q_1, x_1) - \frac{a}{\Phi})}{p_G(1-\lambda)}$  if  $s(q_1, x_1) \leq s(q_2, x_2)$
- $f(q_1, x_1, q_2, x_2) = \frac{s(q_1, x_1) - \frac{1}{2}(s(q_2, x_2) + \frac{a}{\Phi})}{p_G(1-\lambda)}$  otherwise

where  $s(q, x) = \frac{\lambda p_G}{\lambda + (1-q)(1-\lambda)x}$ .

Although in this case RA1 does have reputational concerns, these are not sufficient to prevent RA1 from being lax and not giving GR to bad projects. Since by being honest RA1 is giving up  $I$  today, in exchange for having a higher chance of getting  $I$  in the next period, it is not optimal for RA1 to be honest, given that RA1 is impatient (*i.e.*  $\delta < 1$ ). Hence the optimal strategy of RA1 is to always lie, *i.e.*  $x_{T-1} = 1$ .

□

## A.5 Finite Horizon Solution

We assume the model only lasts for three periods,  $t = 1, 2, 3$ , and the RAs maximise their expected total income over the three periods. We compute the equilibrium strategy of the RAs using backward induction. We already know that the strategic RA will always lie in the last two periods, as shown in Corollary 2.

We solve for the equilibrium strategy at  $t = 1$ . Again, let's look at the decision of RA1. Since RA1 will always lie at  $t = 2, 3$ , the expected pay-off of RA1 at  $t = 1$  is

$$\begin{aligned}\Psi(lie) = & I + \delta V_2(q_1^F, q_2) = I + \delta f(q_1^F, 1, q_2, 1)I + \delta^2 \left( f(q_1^F, 1, q_2, 1)[\lambda p_G f(q_1^F, 1, q_2, 1) \right. \\ & + ((1 - p_G)\lambda + (1 - \lambda))f(q_1^{FF}, 1, q_2, 1)] + f(q_2, 1, q_1^F, 1)[\lambda p_G f(q_1^F, 1, q_2, 1) \\ & \left. + (\lambda(1 - p_G) + (1 - \lambda)(1 - q_2))f(q_1^F, 1, q_2^F, 1) + (1 - \lambda)q_2 f(q_1^F, 1, q_2^N, 1)] \right) I \quad (13)\end{aligned}$$

if it lies, and

$$\begin{aligned}\Psi(honest) = & \delta V_2(q_1^N, q_2) = \delta f(q_1^N, 1, q_2, 1)I + \delta^2 \left( f(q_1^N, 1, q_2, 1)[\lambda p_G f(q_1^N, 1, q_2, 1) \right. \\ & + ((1 - p_G)\lambda + (1 - \lambda))f(q_1^{NF}, 1, q_2, 1)] + f(q_2, 1, q_1^N, 1)[\lambda p_G f(q_1^N, 1, q_2, 1) \\ & \left. + (\lambda(1 - p_G) + (1 - \lambda)(1 - q_2))f(q_1^N, 1, q_2^F, 1) + (1 - \lambda)q_2 f(q_1^N, 1, q_2^N, 1)] \right) I \quad (14)\end{aligned}$$

if it is honest.

As described in Section 3, we look for a Nash equilibrium of the game by examining the trade-off facing RA1, *i.e.* the difference between expressions (13) and (14). If the pay-off from lying is greater then  $x_1 = 1$  and we have a pure-strategy equilibrium in which RA1

always lies; if the pay-off from not lying is greater then  $x_1 = 0$  and we have a pure-strategy equilibrium in which RA1 never lies; otherwise we have a mixed-strategy equilibrium in which RA1 is indifferent between lying and not lying, given some prior beliefs about its strategy, *i.e.*  $0 < x_1 < 1$ .

To derive an analytical solution to this game, we make a simplifying assumption that  $p_G = 1$ . This assumption implies that the reputation of the strategic RA goes to zero if it gives a GR to a bad project since now every good project succeeds and every bad project fails. This simplifies expressions (13) and (14) and allows us to derive the equilibrium strategy of RA1.

The expression of market share of RA1 depends on whether RA1 has a higher probability of success than its competitor. Given that the strategy of the strategic RA in the last two periods is to always lie, the RA with a higher reputation will have a higher market share in any single period. Hence we compute the strategy of RA1 in different ranges of the reputation of RA2.

**Proposition 4.** *The equilibrium strategy at  $t = 1$  assuming  $p_G = 1$  is*

$$x_1 = \begin{cases} 0 & \text{if } A \leq \frac{\lambda q_1}{2(\lambda q_1 + (1 - q_1))} \\ 1 - \frac{(1 - 2A)\lambda q_1}{2A(1 - q_1)} & \text{if } \frac{\lambda q_1}{2(\lambda q_1 + (1 - q_1))} < A < \frac{1}{2} \\ 1 & \text{if } A \geq \frac{1}{2} \end{cases}$$

where  $A$  is the solution to the equation

$$\Psi(lie) - \Psi(honest) = I - \delta(2A - \min\{A, B\})I - \delta^2\left(\lambda(2A - \min\{A, B\})^2 + (2B - \min\{A, B\})[\lambda(2A - \min\{A, B\}) + 2(1 - \lambda)(1 - q_2)A + (1 - \lambda)q_2A]\right)I = 0$$

$$\text{and } B = \frac{\frac{1}{2}\left(s(q_2, 1) - \frac{a}{\Phi}\right)}{p_G(1 - \lambda)}.$$

*Proof.* Since  $p_G = 1$ , the reputation of RA1 (*i.e.* the strategic RA) will go to zero if it gives a GR to a bad project since now every good project succeeds and every bad project fails. So the expected pay-off from giving a GR to a bad project is  $I$ . This simplifies expressions (13) and (14) and allows us to derive RA1's equilibrium strategy.

The expected pay-off from being honest is

$$\begin{aligned} \Psi(honest) &= \delta f(q_1^N, 1, q_2, 1)I + \delta^2\left(f(q_1^N, 1, q_2, 1)\lambda f(q_1^N, 1, q_2, 1) \right. \\ &\quad \left. + f(q_2, 1, q_1^N, 1)[\lambda f(q_1^N, 1, q_2, 1) + (1 - \lambda)(1 - q_2)f(q_1^N, 1, q_2^F, 1) + (1 - \lambda)q_2f(q_1^N, 1, q_2^N, 1)]\right)I \end{aligned}$$

Using equations (6) and (7) and noting that RA1 will always lie in periods  $t = 2, 3$ , this can be rewritten as

$$\begin{aligned} \Psi(honest) &= \delta(2A - \min\{A, B\})I + \delta^2\left(\lambda(2A - \min\{A, B\})^2 \right. \\ &\quad \left. + (2B - \min\{A, B\})[\lambda(2A - \min\{A, B\}) + 2(1 - \lambda)(1 - q_2)A + (1 - \lambda)q_2A]\right)I \end{aligned}$$

$$\text{where } A = \frac{\frac{1}{2}\left(s(q_1^N, 1) - \frac{a}{\Phi}\right)}{p_G(1 - \lambda)} \text{ and } B = \frac{\frac{1}{2}\left(s(q_2, 1) - \frac{a}{\Phi}\right)}{p_G(1 - \lambda)}$$

The expected pay-off from lying is  $I$ , since the RA's reputation goes to zero

$$\Psi(lie) = I$$

We look for a Nash equilibrium of the game by examining RA1's trade-off between lying and not lying. If the pay-off from lying is greater when  $x_1 = 1$ , we have a pure-strategy equilibrium in which RA1 always lies; if the pay-off from not lying is greater when  $x_1 = 0$ , we have a pure-strategy equilibrium in which RA1 never lies; otherwise we have a mixed-strategy equilibrium in which RA1 is indifferent between lying or not given some prior beliefs about its strategy, *i.e.*  $0 < x_1 < 1$ .

We now solve the equation  $\Psi(lie) - \Psi(honest) = 0$ . We do this in 2 stages. In the first stage, we solve the equation in terms of  $A$  and then using the expression for  $A$ , we solve for the equilibrium value of  $x_1$ .

For  $A < B$  we have

$$\Psi(lie) - \Psi(honest) = \delta^2(1 - \lambda)(2 - q_2)A^2 - (\delta + 2B\delta^2\lambda + 2B\delta^2(1 - \lambda)(2 - q_2))A + 1$$

The solution is

$$A = B + \frac{\delta + 2B\delta^2\lambda - \sqrt{(\delta + 2B\delta^2\lambda)^2 + \varrho}}{2\delta^2(1 - \lambda)(2 - q_2)}$$

which is valid<sup>21</sup> as long as  $\varrho = B^2\delta^2(2 - (1 - \lambda)q_2) + \delta B - 1 > 0$ .

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<sup>21</sup>*i.e.* consistent with our assumption that  $A < B$ .

Now for  $A \geq B$  we have

$$\Psi(lie) - \Psi(honest) = -4\delta^2\lambda A^2 - (2\delta - 2B\delta^2\lambda + B\delta^2(1 - \lambda)(2 - q_2))A - \delta B - 1$$

The solution is

$$A = \frac{\sqrt{(B + (2\delta + B\delta^2(1 - \lambda)(2 - q_2)))^2 - \varrho - (2\delta + B\delta^2(1 - \lambda)(2 - q_2))}}{8\delta^2\lambda}$$

which is valid<sup>22</sup> given  $\varrho = B^2\delta^2(2 - (1 - \lambda)q_2) + \delta B - 1 \leq 0$ .

Hence we show that there always exists a solution which depends on the parameter  $\varrho$ . Since  $A$  always has a solution, we can use it to find the equilibrium strategy  $x_1$  in terms of  $A$ , *i.e.*

we will look for the value of  $x_1$  such that  $\frac{\frac{1}{2}(s(q_1^N, 1) - \frac{\varrho}{\Phi})}{p_G(1 - \lambda)} = A$ .

Note that assuming  $p_G = 1$  implies  $\frac{\varrho}{\Phi} = \lambda$ . Using this and equation (5), the above expression can be rewritten as  $\frac{\lambda q_1^N}{\lambda q_1^N + 1 - q_1^N} = 2A$ , where  $q_1^N = \frac{q_1}{1 - (1 - q_1)x_1}$ .

Solving, we obtain

$$x_1 = 1 - \frac{(1 - 2A)\lambda q_1}{2A(1 - q_1)}$$

for  $0 < x_1 < 1$ . This holds when  $\frac{\lambda q_1}{2(\lambda q_1 + (1 - q_1))} < A < \frac{1}{2}$ .

□

Proposition 4 implies that the strategy of RA1 depends on its own and its competitor's reputation. When  $A$  is large, RA1 always gives a GR to a bad project. Conversely, when  $A$  is small RA1 behaves honestly and gives NR to bad projects. In the intermediate range, RA1

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<sup>22</sup>*i.e.*  $A \geq 0$ .

has a mixed strategy, with  $0 < x_1 < 1$ . Note that the lower threshold for A is increasing with RA1's reputation.

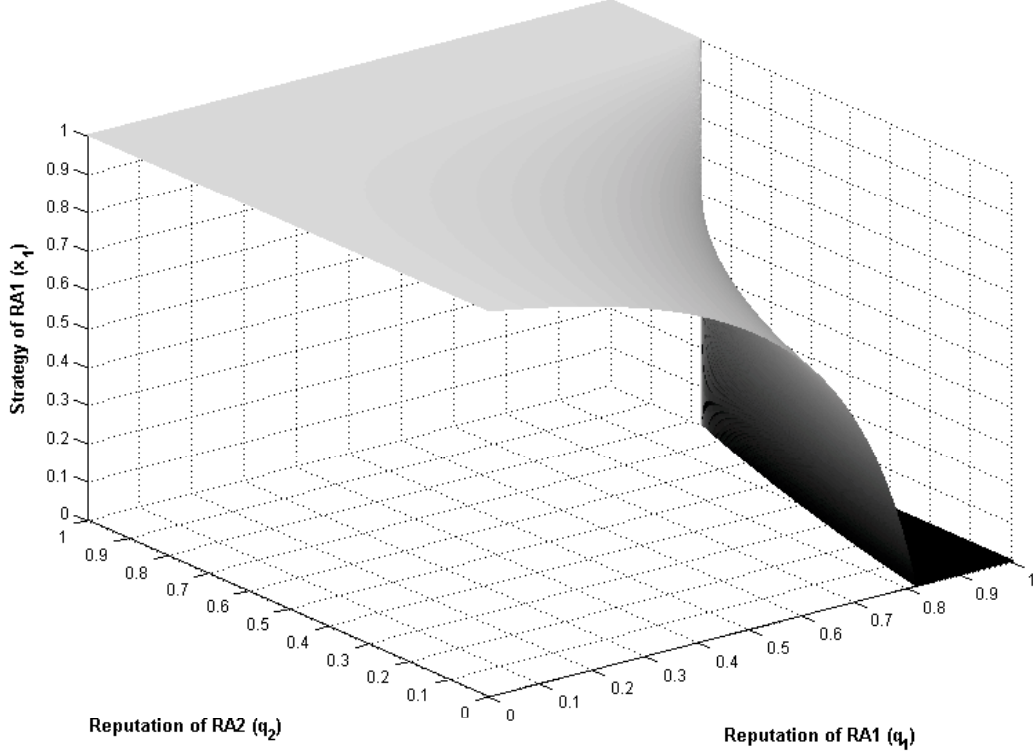


Figure 14: Strategy against reputation ( $\lambda = 0.7$ ,  $\delta = 0.9$ )

Figure 14 plots RA1's strategy against the reputation of RA1 and RA2. We can see that RA1 tends to lie less as its reputation increases, and it tends to lie more as the reputation of RA2 increases.

The intuition behind this result is straightforward. Since we assumed  $p_G = 1$ , the reputation of RA1 goes to zero immediately after a project fails. This means that the cost of lying increases with RA1's reputation while the benefit of lying stays constant. Hence it is not surprising that RA1 prefers to lie less as its reputation increases. On the other hand, the higher the reputation of RA2, the less expected future income RA1 has, and the market-sharing effect

becomes stronger and dominates the disciplining effect. Therefore RA1 would like to inflate ratings when RA2 has a higher reputation.

Note that in this special case (when  $p_G = 1$ ), the RA's strategy is concave in reputation. This stems from the fact that with  $p_G = 1$ , the strategic RA is caught immediately after the project fails and thus the cost of ratings inflation increases with reputation. However, our results in section 4 clearly show that this is no longer true if  $p_G < 1$ . The penalty on reputation will be smaller as the reputation of RA increases, *i.e.* the cost of ratings inflation can decrease with reputation, resulting in a 'u-shaped' relationship between strategy and reputation.



# Credit Default Swaps - Default Risk, Counter-party Risk and Systemic Risk

Zijun Liu

## **Abstract**

Credit Default Swaps can contribute to systemic risk in two ways: through counter-party risk and through sharing of default risks. A central clearing house, which can only reduce counter-party risk, is by no means a panacea. Moreover, excessive risk taken by one reckless institution may spread to the entire financial system via the CDS market, even in the absence of counter-party risk. This could potentially explain the US government's decision to bail out AIG during the recent financial crisis. Policies mitigating the information asymmetry between the market and regulators would be desirable.

**Keywords:** Credit Default Swaps, Systemic Risk, Financial Crisis

**JEL Classifications:** D6, D82, G21, G28

# 1 Introduction

Credit Default Swaps are swap contracts in which the buyer makes a series of payments and receives a payoff if the underlying instrument suffers a credit event, e.g. default. Prior to 2008, the CDS market was over-the-counter, largely unregulated and opaque. The size of the CDS market has grown phenomenally from a few billion dollars in 2003 to around \$45 trillion in 2007. During that time, many including Alan Greenspan argued the CDS market enhanced risk-sharing and helped stabilize the financial system<sup>1</sup>.

In early 2008, Lehman Brothers suffered severe losses on its exposure to sub-prime mortgage securities, and AIG revealed a huge amount CDS protection sold on CDOs and other debt instruments. Lehman Brothers filed for bankruptcy on 15 September, 2008, and AIG was bailed out subsequently by the U.S. government on 16 September, 2008. In the aftermath of the crisis, the CDS market was widely blamed for its contribution to systemic risk through counter-party risk. In response, the regulators have since then been calling for centralized clearing of CDS contracts. Several central clearing houses have already been established in the US and the Europe and over 80% of eligible CDS contracts among dealers are now being cleared through the clearing houses<sup>2</sup>.

This paper shows that the CDS market can pose significant threats to financial stability, and a central clearing house, albeit beneficial, is by no means a panacea. This is because there are two ways through which the CDS market can contribute to systemic risk. First, systemic risk can arise through counter-party risk. When a large dealer defaults, its counter-parties may become more likely to default due to the domino effect. This has been seen as the main risk associated with CDS<sup>3</sup> and will be eliminated by a central clearing house (assuming that the clearing house never defaults). Second, the payments on a CDS contract (often large) are triggered when the reference entities default, which inevitably increases the correlation among financial institutions. That is, when a large amount of risk is shared among a small number

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<sup>1</sup>See <http://www.federalreserve.gov/Boarddocs/Speeches/2005/20050505/default.htm>.

<sup>2</sup>See FSA (2009).

<sup>3</sup>See, for example, ECB (2009).

of banks, the probability that any one bank fails decreases but the probability that all banks jointly fail increases. A central clearing house cannot eliminate the latter type of systemic risk.

The crucial assumption underlying the above argument is that banks become more likely to default when selling CDS contracts. This is because CDS dealing is largely dominated by a small number of dealers (consisting of global investment banks) with substantial exposure. A recent survey by Fitch indicate that 96% of credit derivatives exposures at the end of Q1 2009 of one hundred surveyed firms was concentrated to JP Morgan, Goldman Sachs, Citigroup, and Morgan Stanley and Bank of America<sup>4</sup>. Currently (as of May 2010), the *net* amount of CDS contracts sold by dealers registered on the Depository Trust & Clearing Corporation (DTCC) is close to \$234 billion, and the total *net* notional of single name contracts amounts to \$1200 billion<sup>5</sup>. Moreover, the top ten reference entities account for over 10 % of the total net notional of single names<sup>6</sup>. This implies that the risk in the CDS market is not well-diversified and is shared among a few dealer banks<sup>7</sup>.

More importantly, when an institution becomes reckless and willing to take excessive risk, its trading counter-parties may realize the amount of risk it has taken and profit from buying CDS protection on the reckless institution. Other market participants cannot observe or infer such behaviour, since the CDS market is opaque<sup>8</sup> and the clearing house only clears a limited fraction of CDS trades<sup>9</sup>. Hence, excessive risk taken by one institution can spread to the entire financial system via the CDS market, even in absence of counter-party risk.

This could potentially explain why the US government decided to rescue AIG but not Lehman Brothers during the financial crisis. Out of the \$ 62.1 billion CDS payouts, AIG paid out over a half to just two investment banks: Goldman Sachs and Société Générale. A

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<sup>4</sup>See ECB (2009)

<sup>5</sup>The total net notional of single name contracts may exceed the net notional of CDS contracts sold by dealers, because the position of different single name CDSs do not offset each other in the former.

<sup>6</sup>This is perhaps not surprising, as the hedging demand of the investors tends to be concentrated and proportional to the scale of exposure.

<sup>7</sup>Duffie (2010) has explained that dealer banks are subject to various types of runs in adverse financial conditions. This would make the dealer banks more prone to failures.

<sup>8</sup>Opaqueness means that CDS trades are unobservable not only to other market participants but also to regulators. See Dickinson (2008).

<sup>9</sup>The fact that non-standard contracts and contracts between dealers (banks) and non-dealers are not currently cleared through the central clearing houses is key to this result. See FSA (2009).

reckless manager of a London subsidiary of AIG was responsible for engaging in those trades. Through trading with AIG, Goldman and other counter-parties might have possessed private information about the default risk of AIG and hence had the incentive to actively buy CDS protection on AIG from other financial institutions who were unaware of AIG's exposure. Hence AIG's bankruptcy would have significantly threatened financial stability, given that many financial institutions have sold CDS protection on AIG.

I use a discrete-time model with two periods to illustrate the above intuition. There are three institutions: two banks and one firm. The firm can finance its project by borrowing from one of the banks, who will be exposed to the default risk of the firm. Hence it is optimal for the exposed bank to hedge its risk with the other bank through CDS contracts. In addition, a central clearing house may be established to clear the CDS contracts. Finally, I introduce a risk-averse social planner to capture the negative social consequences associated with systemic risk.

In the baseline case, I find that a CDS market without a central clearing house would significantly increase systemic risk due to counter-party risk, and it is not optimal to have a CDS market if the social planner is risk-averse enough. A central clearing house would greatly reduce systemic risk by eliminating counter-party risk. However, a small amount of systemic risk remains after introducing the central clearing house, since banks have uncertain income and will inevitably face the risk of bankruptcy by selling CDS contracts.

I then assume that, with a certain probability, the firm can become reckless and willing to take on excessive risk, which is not observable by other institutions. In this case, one informed bank will induce the reckless firm to take excessive risk and buy protection on the reckless firm from the other bank, who cannot infer whether the firm is reckless or not. Such behaviour will further contribute to systemic risk. Policies mitigating the information asymmetry between the market and regulators would be desirable.

The rest of the paper is organized as follows. Section 2 is a brief review of related literature. Section 3 presents the model. Section 4 discusses policy implications, Section 5 describes the case of AIG, and Section 6 concludes.

## 2 Related Literature

This paper is closely related to the literature on financial contagion. Earlier studies have focused on financial contagion arising from cross-holding of deposits among banks. Allen and Gale (2000) assume imperfect correlation of liquidity preferences across regions and show that an unexpected small liquidity preference shock in one region can spread throughout the economy. Dasgupta (2004) show that cross-holding of deposits can lead to financial contagion with a positive probability in equilibrium. Instead of the above, this paper explores contagion effects arising from OTC derivative contracts.

Zawadowski (2010) analyzes financial contagion through counter-party risk in bilateral hedging contracts. The paper shows that banks may not buy insurance on the failure of their counter-parties even if it is socially optimal, because banks themselves do not take into account the failure of other banks, hence the failure of one bank can lead to the collapse of the entire financial system. Such effects have been empirically verified by a number of papers, e.g. Jorion and Zhang (2009). Counter-party risk as a channel of contagion is confirmed in but not the main focus of this paper.

Other channels of contagion have been explored. For example, Acharya and Yorulmazer (2002) argue that banks tend to default together because they take correlated investments ex-ante due to herding motives. In Diamond and Rajan (2005), contagion arises from an aggregate liquidity shock and its impacts on the common market for liquidity among banks. Adrian and Shin (2008) find that financial contagion can occur in non-traditional ways based on measured risks and marked-to-market accounting.

A growing literature focuses on the formation and implication of financial networks. Allen and Babus (2008) provide a comprehensive survey of the literature. Babus (2009) model the decision of banks to ex-ante commit to ensure each other against the risk of contagion using a network formation game approach. Babus (2009) show that when banks endogenously form networks to respond to contagion risk, financial stability will be greatly supported.

Cont et al. (2009) models financial contagion using a network approach and proposes a mea-

sure of systemic risk impact taking into account the financial institution's connectivity, magnitude of exposure and correlation of liquidity shocks. Moreover, they pointed out that CDS can generate contagion in non-traditional ways as AIG sold protection on Lehman Brother's debt and needed a bailout after the investment bank went bust. However, they did not explore the incentives of an informed bank to induce the reckless institution to take excessive risk.

Another related literature looks at CDS and bank monitoring incentives. For example, Parlour and Winton (2009) argue that the CDS market reduces the bank's incentives to monitor its loans, and Bolton and Oehmke (2010) show that CDS have important ex-ante commitment benefits but can also lead to excess insurance in equilibrium, which may be overcome by enhancing disclosure. A common ingredient of the above papers and mine is that the bank can trade and change the fundamental value of the asset at the same time. Nevertheless, my paper focuses on the interaction between two agents that generates excessive risk and undermines financial stability.

A number of papers have focused on the implication of central clearing houses. Acharya and Bisin (2009) show that parties in an OTC market may take excess leverage when agent's trades are not mutually observable, and that a centralized exchange would help prevent the build-up of excessive exposures and lead to Pareto-efficient equilibria. Duffie and Zhu (2010) show that having a central clearing house for one class of derivatives only may actually reduce netting efficiency, since central clearing would reduce bilateral netting opportunities. Moreover, Pirrong (2009) argue that a clearinghouse would reduce the incentives of market participants to monitor each other. This paper shows that a central clearing house is indeed beneficial, but cannot curb the risk-taking behaviour of the reckless institution under current circumstances.

Other aspects of the CDS market have also been widely studied. Stulz (2009) discussed the role the CDS market has played during the recent credit crisis and assessed various policy options. Biais et al. (2010) show that credit default swaps creates systemic risk upon arrival of bad information about the protection seller, who would then exert less effort to manage its balance sheet risk, hence increasing the counter-party risk of the protection buyer.

### 3 The Model

#### 3.1 Basic Setup

##### Agents

The model has three dates,  $t = 0, 1, 2$ . We refer to  $t = 1$  as *short-term* and  $t = 2$  as *long-term*. There is a single good in the economy that we can think of as cash. There are three institutions: two *banks* denoted by bank A and bank B, and a *firm*. The banks represent the large investment banks that are dealers in the CDS market, and the firm could be any financial or non-financial institution eligible to trade CDS contracts. In addition, there is a continuum of investors endowed with a finite amount of wealth at  $t = 0$ .

All institutions are protected by limited liability and maximize shareholder value<sup>10</sup> at  $t = 2$ . However, the investors have uncertain date of consumption. With probability  $\frac{1}{2}$ , the investors will suffer from a *liquidity shock* and consume at  $t = 1$ , and with probability  $\frac{1}{2}$  the investors consume at  $t = 2$ .<sup>11</sup> The arrival date of the liquidity shock is unknown ex-ante. All agents are risk-neutral.

##### Projects

Each bank can invest in a *fixed-scale* project at  $t = 0$  with an initial investment of  $I$  and cash flow  $R$  at  $t = 1$  and  $V$  at  $t = 2$ , where  $V$  is certain but  $R$  is random and can be  $I$  with probability  $\lambda$  and  $I + r$  with probability  $1 - \lambda$  (assume  $r > 0$ ). We refer to  $R = I + r$  as *high income* and  $R = I$  as *low income*. We can think of  $V$  as the long-term value of the institution which is large relative to the short-term income  $R$ . The realization of each bank's short-term income is only observable by the bank itself and not to other agents.

The firm also has a similar investment opportunity with initial investment  $I$ , albeit much riskier, in the sense that the return of the project can be negative. Specifically, with probability

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<sup>10</sup> Assume there are no agency problems between the management and the shareholders of each institution.

<sup>11</sup> Investors are assumed to have perfectly correlated liquidity shocks and consume at  $t = 1$  or  $t = 2$  with equal probabilities. Such assumptions are for the purpose of simplicity only and does not change the qualitative results of the model.

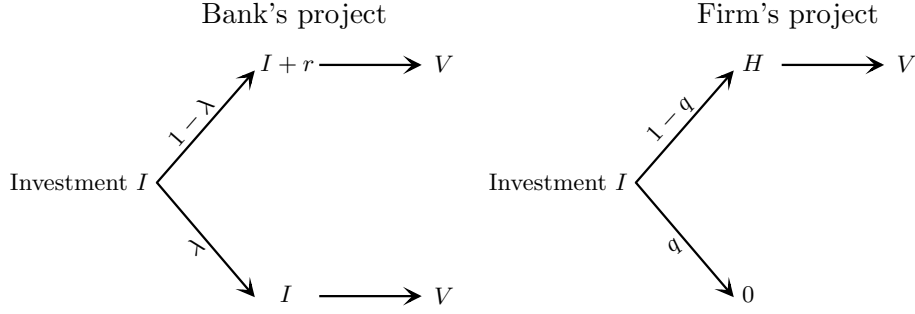


Figure 1: Project payoffs of the banks and the firm

$1 - q$  the project succeeds with payoff  $H$  at  $t = 1$  and  $V$  at  $t = 2$ , and with probability  $q$  the project fails with payoff 0 at  $t = 1$ . Assume  $H > \frac{I}{1-q}$ , so that the project has a positive NPV in the short term. The realization of the project's short-term payoff is only observable by the firm. The cash flows on all projects are independent across institutions and illustrated in Figure 1.

The investors cannot directly invest in those projects. In addition, all agents have access to a storage technology with rate of return equal to zero.

## Financing

These institutions do not have any initial wealth. However, the banks can issue *short-term debt*<sup>12</sup> to the investors. The investors are competitive and have sufficient wealth to finance the projects. Moreover, the firm can borrow from one of the banks<sup>13</sup>. The firm cannot issue debt to the investors, which can be potentially explained by asymmetric information about the quality of the firm. Without loss of generality, I assume the firm borrows from bank A throughout the paper.

If any institution fails to repay its debt, it will become bankrupt and its assets will be liquidated. This is a publicly observable event. Assume that the debt holders have the highest

<sup>12</sup>I rule out long-term financing by assuming that the institutions are able to divert funds in the long-term in a non-verifiable way. Short-term debt can be an extremely powerful tool to monitor management, as argued in Stulz (2000). Similar assumptions have been used in, for instance, Diamond and Rajan (2001) and Martin et al. (2010).

<sup>13</sup>I assume that the firm cannot borrow from two banks at the same time, possibly due to client relationships, geographical locations and other constraints.



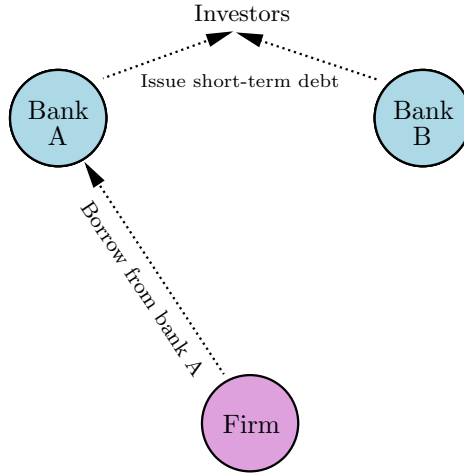


Figure 2: Basic setup

priority in the event of bankruptcy. The liquidation value of any project is 0 (in addition to current-period income).

Recall that the investors will consume early if they suffer a liquidity shock. The state of liquidity shock can be interpreted as a credit crunch in which borrowing becomes extremely difficult for the banks. Hence there will be no bankruptcy if there is no liquidity shock at  $t = 1$ , since the banks will then be able to refinance or renegotiate with the investors so that the projects may be continued.

The short-term income of all institutions are only observable by themselves, but can be verified by another party after a verification process. Assume that the short-term income of an institution is only verifiable by its creditors, after the institution has defaulted. This has two implications: firstly, no institutions would strategically default when they can fulfill their obligations; secondly, the CDS contract cannot be made contingent on the seller's short-term income.

Many studies have shown under certain conditions that when information is asymmetric and verification is costly, the optimal contracts are debt contracts such that verification by the creditors only happens in the default state<sup>14</sup>. Similarly, the assumption above can be

<sup>14</sup>See, for example, Townsend (1979) and Williamson (1986).



Note that the long-term output of the firm is also taken into account by the social planner. This is because the firm is also part of the financial system and its failure would also threaten financial stability, for instance AIG, through the service it provides or its links to the financial system. Such an institution is non-bank in this paper in which exposure is modeled as bank lending, but could be any bank or non-bank institution in reality given other forms of exposure such as OTC derivatives.

Suppose we have a risk-averse social planner who maximizes the his expected utility over the total output (the payoff from all projects) at  $t = 1$  and  $t = 2$ . Because all projects will be financed at  $t = 0$  as shown later, the social planner cannot influence the output at  $t = 1$ . Hence the social planner's decision is effectively maximizing total output at  $t = 2$ . In the rest of the paper, we are going to assume that the social planner maximizes the expected value of the function  $f(Y)$ , where  $Y$  denotes the total output at  $t = 2$  and  $f$  is a strictly increasing and concave function of  $Y$  with  $f(0) = 0$ .

The rest of the model is presented in two sections. The baseline equilibrium is shown in Section 3.2, and a reckless firm is introduced in Section 3.3.

## 3.2 Baseline Case

In the following, I focus on the CDS market and its impact on systemic risk. In equilibrium, CDS contracts are optimal contracts among other trivial variations. Therefore, through out the paper, I label the case in which institutions cannot contract with each other as “no CDS market”, and vice versa. We first look at the case without CDS market as a benchmark, and the details of CDS contracts will be explained later.

### 3.2.1 No CDS Market

Suppose there is no CDS market, i.e. the institutions cannot write contract with each other except loan contracts. Debt of amount  $I$  is required for each bank to invest in its own project, which is riskless since the short-term payoff is greater or equal to  $I$ . In addition, bank A

needs to raise extra debt in order to lend  $I$  to the firm. Let's denote the notional amount (the amount to be repaid) of extra debt issued by bank A by  $N^A$ . Because the payoff from the project of the firm is not contractible (it is only verifiable when the firm defaults and the creditors have full control), the contract between bank A and the firm will be a simple loan contract with a fixed notional amount, denoted by  $N^F$ , to be repaid at  $t = 1$ . Finally, assume bank A is competitive when lending to the firm, since the firm can potentially borrow from bank B instead.

When the firm defaults on the loan at  $t = 1$ , bank A may become bankrupt if refinancing is not possible. In this case, we say that bank A is *exposed* to the firm. Note that all institutions are concerned about bankruptcy risk despite limited liability, since bankruptcy would imply early liquidation of projects and loss of their long-term payoff  $V$ .

Banks have uncertain short-term income which is either  $I + r$  or  $I$ , so  $r$  may be interpreted as the “free cash flow”. Suppose bank A lent to the firm who then defaulted at  $t = 1$ , and refinancing is not possible (due to the liquidity shock). The notional amount of extra debt issued by bank A, denoted by  $N^A$ , satisfies  $I \leq N^A \leq \frac{I}{1-q}$  given that the investors are competitive. If  $r$  is too large, bank A would be able to avoid bankruptcy if it had high income. If  $r$  is too small, even the total income of bank A and B together will not be enough to repay bank A's debt. In order to focus on the case where risk-sharing is Pareto-optimal, we assume  $\frac{I}{2(1-q)} < r < I$ , i.e. the size of the potential income of the banks is such that bank A can never withstand the failure of the firm alone, but may potentially do so if it hedged its exposure with bank B. This assumption is only valid given  $q < \frac{1}{2}$ , as in Assumption 3.1.

**Assumption 3.1.** Assume  $\frac{I}{2(1-q)} < r < I$  and  $q < \frac{1}{2}$ .

In addition, we assume  $V \geq \frac{I}{1-q}$ , so that the debt can be fully refinanced if there is no liquidity shock<sup>17</sup>. In addition, we assume  $H \geq \frac{I+\frac{q}{2}V}{1-q}$ . Recall that  $H$  is the payoff of the firm's project if it succeeds. If bank A lends to the firm, the probability that bank A becomes bankrupt at  $t = 1$  would increase from 0 to  $\frac{q}{2}$ . This assumption ensures that bank A is

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<sup>17</sup>This assumption simplifies the analysis of the model. Assuming a smaller  $V$  would imply that the price of the debt is dependent on  $V$ . The qualitative results of the model still hold in this case.

willing to lend to the firm and able to repay its debt if the firm's project is successful. These assumptions are set out in Assumption 3.2.

**Assumption 3.2.** Assume  $V \geq \frac{I}{1-q}$  and  $H \geq \frac{I + \frac{q}{2}V}{1-q}$ .

**Proposition 3.1.** If there is no CDS market, the following is a Nash equilibrium given Assumption 3.1 and 3.2. At  $t = 0$ :

*Bank A issues debt with price  $2I$  and notional amount  $I + \frac{I - \frac{q}{2}(1-\lambda)r}{1 - \frac{q}{2}}$ , invests in its own project and lends  $I$  to the firm. The notional value of the loan is  $N^F = \frac{I + \frac{q}{2}V}{1-q}$ .*

*Bank B issues debt with price  $I$  and notional amount  $I$  and invests in its own project*

*Proof.* See Appendix A.1. □

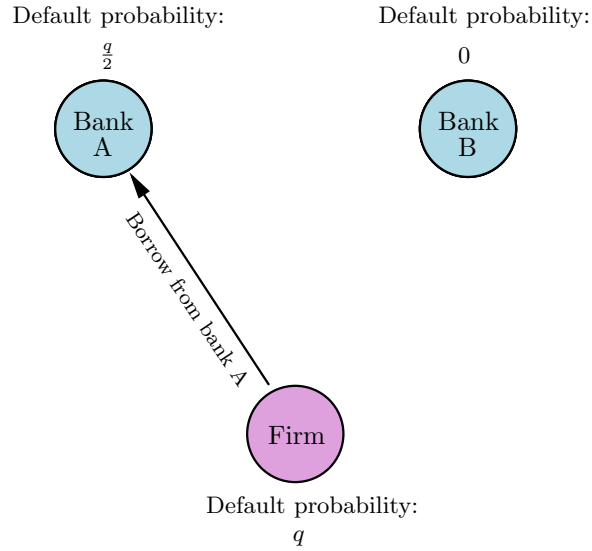


Figure 4: Equilibrium without CDS Market

The equilibrium is shown in Figure 4. Bank A lends to the firm and raises  $2I$  from the investors, whereas bank B only raises  $I$ . Because the investors are competitive, the notional amount of debt is such that the expected return on the debt is equal to zero. At  $t = 1$ , the investors of bank A will be fully repaid with probability  $1 - \frac{q}{2}$ , or receive  $\mathbf{E}(R) = I + (1 - \lambda)r$  otherwise, hence the notional amount of bank A's debt is  $I + \frac{I - \frac{q}{2}(1-\lambda)r}{1 - \frac{q}{2}}$ . Given that bank A is

competitive when lending to the firm, the notional value of the loan is  $N^F = \frac{I + \frac{q}{2}V}{1-q}$ , reflecting the potential loss of  $V$  due to the exposure.

**Corollary 3.1.** *If there is no CDS market, we have*

$$\mathbf{P}(Y = 3V | \text{No CDS}) = 1 - q$$

$$\mathbf{P}(Y = 2V | \text{No CDS}) = \frac{q}{2}$$

$$\mathbf{P}(Y = V | \text{No CDS}) = \frac{q}{2}$$

$$\mathbf{P}(Y = 0 | \text{No CDS}) = 0$$

*Proof.* Follows from Proposition 3.1. □

Corollary 3.1 describes the probability distribution of the output at  $t = 2$ . We can see that the probability that two institutions default together ( $Y = V$ ) is  $\frac{q}{2}$ , since the firm defaults with probability  $q$  and conditionally bank A defaults with probability  $\frac{1}{2}$ . Moreover, the probability that all three institutions default is zero in this case, since only one bank can be exposed to the firm and no inter-bank hedging takes place.

### 3.2.2 CDS Market

Now suppose contracts can be written among the institutions<sup>18</sup> at  $t = 0$ , particularly CDS contracts. I define a CDS contract as a bilateral contract in which the seller pays the notional amount upon realization of a credit event at  $t = 1$ , and the buyer pays a premium at  $t = 2$  otherwise. We will show that the CDS contract is the optimal contract in equilibrium<sup>19</sup>.

For simplicity, I assume there are no collateral requirements. In reality, the level of collateralization of CDS is typically a function of the market value of the contract, which tends to be much lower than the notional amount, given the small probability of default<sup>20</sup>. As this

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<sup>18</sup>Assume that CDS contracts are sophisticated derivatives that are not available to individual investors.

<sup>19</sup>While there exist other forms of optimal contracts with trivial variations, we focus on CDS contracts for its simplicity and relevance in reality. The results hold for other forms of optimal contracts.

<sup>20</sup>See ECB (2009).

paper focuses on the jump-to-default risk rather than the spread risk of CDS contracts, this seems an innocuous assumption. In addition, I assume that bank B has no bargaining power, i.e. its expected gain is equal to zero<sup>21</sup>.

Suppose the menu of contracts between bank A and bank B is given by a payment  $\tau_i^B$  ( $i = 1, 2$ ) from bank A to bank B at  $t = 1, 2$  respectively (negative values indicate payments in the opposite directions), conditional on all contractible information. Note that the payment cannot be made contingent on banks' short-term income as previously explained, so the only contractible information is the solvency of the firm and the liquidity shock. Let's denote the state that the firm defaults by  $s = \bar{s}$ , and the state that the firm survives by  $s = \underline{s}$ . Similarly, denote the state with a liquidity shock by  $l = \bar{l}$  and the state without a liquidity shock by  $l = \underline{l}$ .

Assuming that bank A has issued extra debt with notional  $N^A$  and lent to the firm with notional  $N^F$ , bank A chooses  $\tau_i^B$  conditional on  $s$  and  $l$  to maximize<sup>22</sup>

$$\begin{aligned} & (1 - \frac{q}{2})((1 - \lambda)r + V) + (1 - q)(N^F - N^A) + \frac{1 - q}{2}(-\tau_1^B(\underline{s}, \underline{l}) - \tau_2^B(\underline{s}, \underline{l})) \\ & + \frac{1 - q}{2}(\mathbf{I}(\tau_1^B(\underline{s}, \bar{l}) = 0)(-\tau_2^B(\underline{s}, \bar{l})) + \mathbf{I}(0 > \tau_1^B(\underline{s}, \bar{l}) \geq -r)((1 - \lambda)(-\tau_1^B(\underline{s}, \bar{l}) - \tau_2^B(\underline{s}, \bar{l})) + \lambda(r - \tau_2^B(\underline{s}, \bar{l}))) \\ & + \mathbf{I}(\tau_1^B(\underline{s}, \bar{l}) < -r)((1 - \lambda)r - \tau_2^B(\underline{s}, \bar{l}))) + \frac{q}{2}(-\tau_1^B(\bar{s}, \underline{l}) - \tau_2^B(\bar{s}, \underline{l})) \\ & + \frac{q}{2}(1 - \lambda)^2 \mathbf{I}(\tau_1^B(\bar{s}, \bar{l}) \leq r - N^A)(-\tau_1^B(\bar{s}, \bar{l}) - (N^A - r) - \tau_2^B(\bar{s}, \bar{l}) + V) \end{aligned}$$

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<sup>21</sup>This assumption implies that there is potentially another bank who is willing to compete against banks B and is for the purpose of simplification only.

<sup>22</sup>Here I assume that  $-V \leq \tau_1^B \leq 0$  and  $-V \leq \tau_2^B \leq V$  in order to avoid unnecessary complications in the expression, see the proof of Proposition 3.2. In addition, the debt holders of bankrupt institutions take over all net claims on contracts.

subject to the participation constraint of bank B

$$\begin{aligned}
& \frac{1-q}{2}(\tau_1^B(\underline{s}, \underline{l}) + \tau_2^B(\underline{s}, \underline{l})) + \frac{1-q}{2}(\mathbf{I}(\tau_1^B(\underline{s}, \bar{l}) = 0)\tau_2^B(\underline{s}, \bar{l}) + \mathbf{I}(0 > \tau_1^B(\underline{s}, \bar{l}) \geq -r) \\
& ((1-\lambda)(\tau_1^B(\underline{s}, \bar{l}) + \tau_2^B(\underline{s}, \bar{l})) + \lambda(-V)) + \mathbf{I}(\tau_1^B(\underline{s}, \bar{l}) < -r)(-(1-\lambda)r - V)) \\
& + \frac{q}{2}(\tau_1^B(\bar{s}, \underline{l}) + \tau_2^B(\bar{s}, \underline{l})) + \frac{q}{2}(\mathbf{I}(0 > \tau_1^B(\bar{s}, \bar{l}) \geq -r)((1-\lambda)^2(\tau_1^B(\bar{s}, \bar{l}) + \tau_2^B(\bar{s}, \bar{l})) \\
& + \lambda(1-\lambda)\tau_1^B(\bar{s}, \bar{l}) - \lambda V) + \mathbf{I}(\tau_1^B(\bar{s}, \bar{l}) < -r)(-(1-\lambda)r - V)) \geq 0
\end{aligned}$$

where  $\mathbf{I}(\cdot)$  is an indicator function that is equal to 1 if the expression  $(\cdot)$  is true, and 0 otherwise.

Intuitively, bank A can hedge its exposure to the firm by contracting with bank B at  $t = 0$ . Because the short-term income of bank A and bank B are uncorrelated, bank B can potentially compensate bank A should the firm default. Hence the optimal contract involves a negative payment  $\tau_1^B$  (from bank B to bank A) conditional on the solvency of the firm at  $t = 1$ , in exchange for a positive payment  $\tau_2^B$  at  $t = 2$ .

Bank A can either fully hedge ( $\tau_1^B = N^A$ ) or partially hedge ( $\tau_1^B = N^A - r$ ) its exposure with bank B. If bank A fully hedges with bank B, bank B will not have enough income to make the CDS payments given Assumption 3.1. Hence bank A will not fully hedge its exposure, because otherwise the default probability of bank A will reduce by less than  $\frac{q}{2}$  and that of bank B will increase by  $\frac{q}{2}$ , leaving no incentives to contract.

However, when bank A only partially hedges, bank B may still default on the CDS contract when it has low income with probability  $\lambda$ . In addition, bank A retains some bankruptcy risk given a partial hedge. Suppose the firm defaults and refinancing is not possible. Then bank B will become bankrupt with probability  $\lambda$ , and bank A will avoid bankruptcy with probability  $(1-\lambda)^2$ . We need to ensure that the net gain from reduced bankruptcy  $((1-\lambda)^2 - \lambda)V$  is large enough<sup>23</sup>. Therefore hedging will only be Pareto-optimal if  $\lambda$  is small enough, as in Assumption 3.3.

**Assumption 3.3.** Assume  $((1-\lambda)^2 - \lambda)V > (1-\lambda)(\frac{I}{1-\frac{q}{2}\lambda} - r)$ .

<sup>23</sup>This is not the exact expression for net gain from hedging, because banks are protected by limited liability. Hence the right-hand-side of the inequality in Assumption 3.3 is not equal to zero.



Note that banks need to take on some bankruptcy risk when selling CDS contracts. This is a key feature of the model. As mentioned in the introduction, trading in the CDS market is concentrated in a small number of dealers (consisting of large investment banks) who are net sellers of CDS protection with substantial exposure. The *net* amount of CDS contracts sold by dealers registered on the DTCC is close to \$234 billion, and the total *net* notional of single name contracts amounts to \$1200 billion. Moreover, the top ten reference entities account for over 10 % of the total net notional of \$1200 billion.

**Proposition 3.2.** *Suppose there is a CDS market. Given Assumption 3.1, 3.2 and 3.3, the following is a Nash equilibrium. At  $t = 0$ :*

*The optimal contract between bank A and bank B is a CDS contract given by*

$$\begin{aligned} \bullet \tau_1^B(\bar{s}) &= r - \frac{I}{1-\frac{q}{2}\lambda} \\ \bullet \tau_2^B(\underline{s}) &= \frac{-q(1-\frac{\lambda}{2})\tau_1^B(\bar{s}) + \frac{q}{2}\lambda V}{1-q} \end{aligned}$$

*Bank A issues debt with price  $2I$  and notional amount  $I + \frac{I}{1-\frac{q}{2}\lambda}$ , invests in its own project and lends  $I$  to the firm. The notional value of the loan is  $N^F = \frac{I + \frac{q}{2}(1-(1-\lambda)^2 + \lambda)V}{1-q}$*

*Bank B issues debt with price  $I$  and notional amount  $I$  and invests in its own project*

*Proof.* See Appendix A.2. □

Figure 5 illustrates the equilibrium. Banks naturally want to avoid bankruptcy, which will result in liquidation of the project and loss of  $V$ . If bank A buys CDS protection on the firm from bank B, its default probability would be reduced to  $\frac{q}{2}(\lambda + \lambda(1 - \lambda))$ , since bank A will default when bank B cannot make the CDS payment or bank A has a low income. The default probability of bank B would increase to  $\frac{q}{2}\lambda$ , since bank B will default when it has a low income. We can see that bank A has incentives to hedge with bank B since the probability of default is a convex function of exposure given that  $\lambda$  is small enough. Bank A only partially hedges its exposure ( $\tau_1^B = N^A - r$ ), since the premium bank A would otherwise have to pay is too high. Hence the bankruptcy risk of bank A is reduced and shared with bank B.

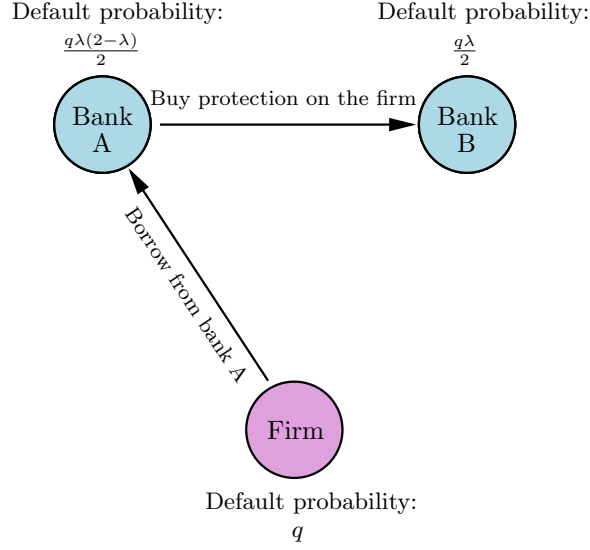


Figure 5: Equilibrium with CDS Market

In fact, the default probability of bank B will increase by at least  $\frac{q}{2}\lambda$  when selling CDS contracts with any positive notional amount, due to the assumption that a bank with low income does not have any free cash. This implies that bank A does not have incentives to hedge with many banks, since the amount of CDS premium to be paid will be too high<sup>24</sup>. This is a result of the simple income structure of the banks assumed in the model and cannot be generalized to reflect the reality. However, the model does predict that when a large amount of risk is shared between a small number of banks, each bank needs to retain a significant portion of risk. This setup is not too unrealistic because CDS dealing is concentrated in a few investment banks as explained before.

Note how counter-party risk has contributed to systemic risk in this case. Suppose the firm defaults and refinancing is not possible at  $t = 1$ . If bank B has a low income, then bank B cannot fulfill its CDS contract with bank A and would default. This implies that bank A will not have enough cash to repay its debt, even if it had a high income, so the failure of bank B

<sup>24</sup>If we allow contracting with multiple banks (with the same income structure), then bank A would not have incentives to hedge with more than one bank if there is a central clearing house. However, when there is no central clearing house, bank A would hedge with exactly two banks given  $2\lambda(2-\lambda) < 1$ , or  $\lambda < 1 - \frac{1}{\sqrt{2}}$ , due to counter-party risk. In order to focus on the implication of CDS market without counter-party risk and to be consistent throughout, I assume a two-bank setting in this section. This assumption is no longer necessary in the following sections.

would also fail bank A. Therefore bank defaults become more correlated due to counter-party risk, as shown in Corollary 3.2.

**Corollary 3.2.** *If there is a CDS market at  $t = 0$ , we have*

$$\begin{aligned}\mathbf{P}(Y = 3V|CDS) &= 1 - q = \mathbf{P}(Y = 3V|No\ CDS) \\ \mathbf{P}(Y = 2V|CDS) &= \frac{q}{2} + \frac{q}{2}(1 - \lambda)^2 > \mathbf{P}(Y = 2V|No\ CDS) \\ \mathbf{P}(Y = V|CDS) &= \frac{q}{2}\lambda(1 - \lambda) < \mathbf{P}(Y = V|No\ CDS) \\ \mathbf{P}(Y = 0|CDS) &= \frac{q}{2}\lambda > \mathbf{P}(Y = 0|No\ CDS)\end{aligned}$$

and  $\mathbf{E}(f|CDS) < \mathbf{E}(f|No\ CDS)$  if and only if  $f(2V) < f(V)(1 + \frac{\lambda}{(1-\lambda)^2})$ .

*Proof.* Follows from Proposition 3.2. □

Figure 6<sup>25</sup> illustrates the impact of the CDS market on systemic risk. As we can see, although the probability that two institutions remain solvent increased after introducing a CDS market, the probability that all institutions default together also increased.

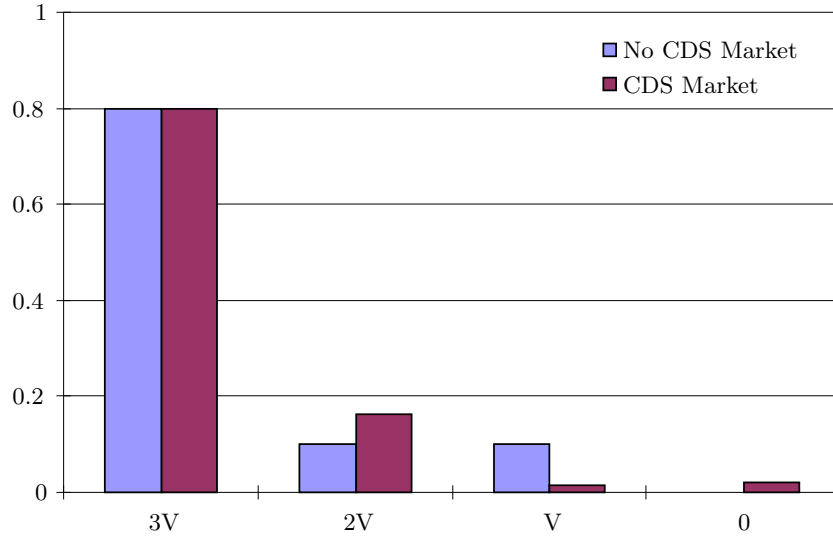


Figure 6: Probability distribution of output at  $t = 2$  with CDS market

<sup>25</sup>Parameter values used throughout this paper are:  $q = 0.2, \lambda = 0.2, \gamma = 0.2, V = 2I$ .

Figure 7 shows the gain/loss of the CDS market without a central clearing house, where risk-aversion is defined to be  $\frac{f(V)}{f(2V)}$ . We can see that the social planner would prefer not to introduce a CDS market given a high enough risk-aversion ( $f(2V) < f(V)(1 + \frac{\lambda}{(1-\lambda)^2})$ ).

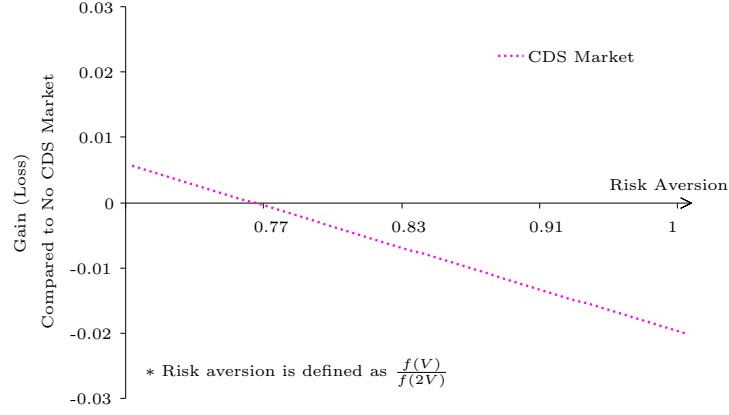


Figure 7: Gain (Loss) of CDS Market

### 3.2.3 CDS Market with Central Clearing House

In this section, a central clearing house is introduced to eliminate counter-party risk. A central clearing house is a financial institution that provides clearing services for CDS transactions. CDS contracts are novated by the central clearing house who act as a counter-party to both the buyer and the seller. For simplicity, I assume that the clearing house is financed by the investors through short-term debt, and the debt holders will be paid after CDS contracts are settled in the event of bankruptcy (so that the clearing house never defaults on CDS contracts). Also assume that the clearing house collects a competitive fee (to be paid at  $t = 2$ ) from the seller (only) when clearing CDS contracts<sup>26</sup>.

Note that the clearing house is financed by outside investors rather than the institutions trading CDS contracts. In practice, a central clearing house typically keeps a default fund to safeguard against the failures of clearing members, using contributions from all clearing members as well as the clearing house itself. In addition, central clearing houses have access to

<sup>26</sup>The clearing arrangements are exogenously given to simplify the analysis and do not affect the qualitative results of the model. Moreover, I assume the clearing fee will be exempt if the clearing house becomes bankrupt beforehand.

credit lines provided by financial institutions and central banks<sup>27</sup>. Hence this assumption is not unrealistic. Modeling a clearing house capitalized by banks leads to the analysis on the trade-off between investment returns and stability. This would bring unnecessary complications as this paper focuses on the limitation rather than the benefits of a clearing house.

Suppose there is a central clearing house at  $t = 0$ . Effectively, if any party in a CDS contract defaults, the clearing house would step in and make the payment. Note that in this model the buyer of CDS contract would never default if the reference entity survives, so that the seller has no counter-party risk. Because hedging gains are greater in absence of counter-party risk, assumptions used in Proposition 3.2 will be sufficient for hedging to take place.

Denote the fee charged by the clearing house by  $z$  and the notional amount of debt issued by the clearing house by  $N^C$ . Assuming that bank A has issued extra debt with notional  $N^A$  and lent to the firm with notional  $N^F$ , bank A will choose  $\tau_i^B$  conditional on  $s$  and  $l$  to maximize

$$\begin{aligned} & (1 - \frac{q}{2})((1 - \lambda)r + V) + (1 - q)(N^F - N^A) + \frac{1 - q}{2}(-\tau_1^B(\underline{s}, l) - \tau_2^B(\underline{s}, l)) \\ & + \frac{1 - q}{2}(\mathbf{I}(\tau_1^B(\underline{s}, \bar{l}) = 0)(-\tau_2^B(\underline{s}, \bar{l})) + \mathbf{I}(0 > \tau_1^B(\underline{s}, \bar{l}) \geq -r)((1 - \lambda)(-\tau_1^B(\underline{s}, \bar{l}) - \tau_2^B(\underline{s}, \bar{l})) + \lambda(r - \tau_2^B(\underline{s}, \bar{l}))) \\ & \quad + \mathbf{I}(\tau_1^B(\underline{s}, \bar{l}) < -r)((1 - \lambda)r - \tau_2^B(\underline{s}, \bar{l}))) + \frac{q}{2}(-\tau_1^B(\bar{s}, l) - \tau_2^B(\bar{s}, l)) \\ & \quad + \frac{q}{2}(1 - \lambda)\mathbf{I}(\tau_1^B(\bar{s}, \bar{l}) \leq r - N^A)(-\tau_1^B(\bar{s}, \bar{l}) - (N^A - r) - \tau_2^B(\bar{s}, \bar{l}) + V) \end{aligned}$$

subject to the participation constraint of bank B

$$\begin{aligned} & \frac{1 - q}{2}(\tau_1^B(\underline{s}, l) + \tau_2^B(\underline{s}, l)) + \frac{1 - q}{2}(\mathbf{I}(\tau_1^B(\underline{s}, \bar{l}) = 0)\tau_2^B(\underline{s}, \bar{l}) + \mathbf{I}(0 > \tau_1^B(\underline{s}, \bar{l}) \geq -r) \\ & \quad ((1 - \lambda)(\tau_1^B(\underline{s}, \bar{l}) + \tau_2^B(\underline{s}, \bar{l})) + \lambda(-V)) + \mathbf{I}(\tau_1^B(\underline{s}, \bar{l}) < -r)(-(1 - \lambda)r - V)) \\ & \quad + \frac{q}{2}(\tau_1^B(\bar{s}, l) + \tau_2^B(\bar{s}, l)) + \frac{q}{2}(\mathbf{I}(0 > \tau_1^B(\bar{s}, \bar{l}) \geq -r)((1 - \lambda)^2(\tau_1^B(\bar{s}, \bar{l}) + \tau_2^B(\bar{s}, \bar{l})) \\ & \quad + \lambda(1 - \lambda)\tau_1^B(\bar{s}, \bar{l}) - \lambda V) + \mathbf{I}(\tau_1^B(\bar{s}, \bar{l}) < -r)(-(1 - \lambda)r - V)) - \frac{1}{2}z \geq 0 \end{aligned}$$

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<sup>27</sup>See CME Group (2009) and Shearman and Sterling LLP. (2009).

where  $z$  is such that the participation constraint of the clearing house binds.

**Proposition 3.3.** *Suppose there is a CDS market with a central clearing house. Given Assumption 3.1, 3.2 and 3.3, the following is a Nash equilibrium. At  $t = 0$ :*

*The optimal contract between bank A and bank B is a CDS contract given by*

- $\tau_1^B(\bar{s}) = r - (I + \frac{q}{2}\lambda r)$
- $\tau_2^B(\underline{s}) = \frac{-q\tau_1^B(\bar{s}) + \frac{q}{2}\lambda V}{1-q}$

*The CDS contract is cleared through the clearing house with a fee  $z = -q\lambda\tau_1^B(\bar{s})$*

*Bank A issues debt with price  $2I$  and notional amount  $2I + \frac{q}{2}\lambda r$ , invests in its own project and lends  $I$  to the firm. The notional value of the loan is  $N^F = \frac{I + q\lambda V}{1-q}$*

*Bank B issues debt with price  $I$  and notional amount  $I$  and invests in its own project*

*The clearing house issues debt with price  $-\tau_1^B(\bar{s})$  and notional amount  $N^C = -\tau_1^B(\bar{s})(1 + q\lambda)$*

*Proof.* See Appendix A.3. □

The equilibrium is shown in Figure 8. As we can see, the equilibrium is the same as before, except that CDS contracts are cleared through the central clearing house and the default probability of bank A is reduced to  $\frac{q}{2}\lambda$ . This is because when bank B defaults, the central clearing house would always be able to make the CDS payments to bank A, who will survive as long as it has high income at  $t = 1$ . Hence counter-party risk is completely eliminated. As a result, the CDS premium paid by bank A is higher than that in the case without a central clearing house.

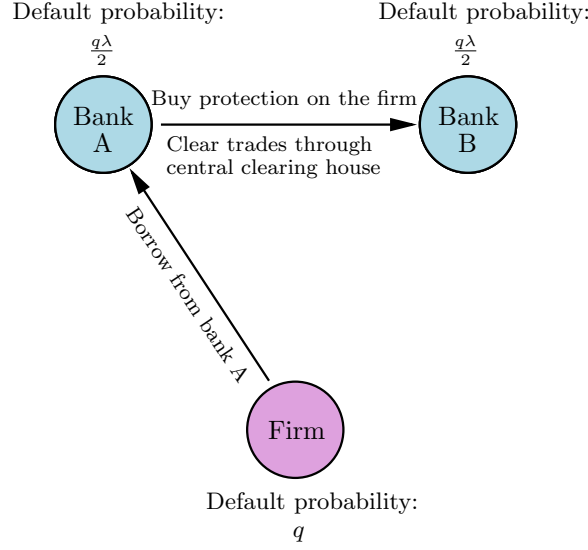


Figure 8: Equilibrium with CDS Market and Central Clearing House

**Corollary 3.3.** *If there is a CDS market and a central clearing house at  $t = 0$ , we have*

$$\begin{aligned} \mathbf{P}(Y = 3V | CDS + Clearing House) &= 1 - q = \mathbf{P}(Y = 3V | CDS) \\ \mathbf{P}(Y = 2V | CDS + Clearing House) &= \frac{q}{2} + \frac{q}{2}(1 - \lambda)^2 = \mathbf{P}(Y = 2V | CDS) \\ \mathbf{P}(Y = V | CDS + Clearing House) &= q\lambda(1 - \lambda) > \mathbf{P}(Y = V | CDS) \\ \mathbf{P}(Y = 0 | CDS + Clearing House) &= \frac{q}{2}\lambda^2 < \mathbf{P}(Y = 0 | CDS) \end{aligned}$$

and  $\mathbf{E}(f | CDS + Clearing House) > \mathbf{E}(f | CDS)$ . Moreover, we have

$$\mathbf{E}(f | CDS + Clearing House) > \mathbf{E}(f | No CDS) \text{ if and only if } f(2V) > f(V)(1 + \frac{\lambda^2}{(1-\lambda)^2}).$$

*Proof.* Follows from Proposition 3.3. □

As shown in Figure 9 <sup>28</sup>, the probability that all banks go bankrupt is much smaller than before, indicating that the central clearing house has greatly reduced systemic risk through the elimination of counter-party risk.

<sup>28</sup>The maximum of the scale is set to 0.4 in order to better illustrate the small changes in probabilities. The probability that  $Y = 3V$  is the same (equal to  $1 - q$ ) for all probability distribution plots in the baseline case.

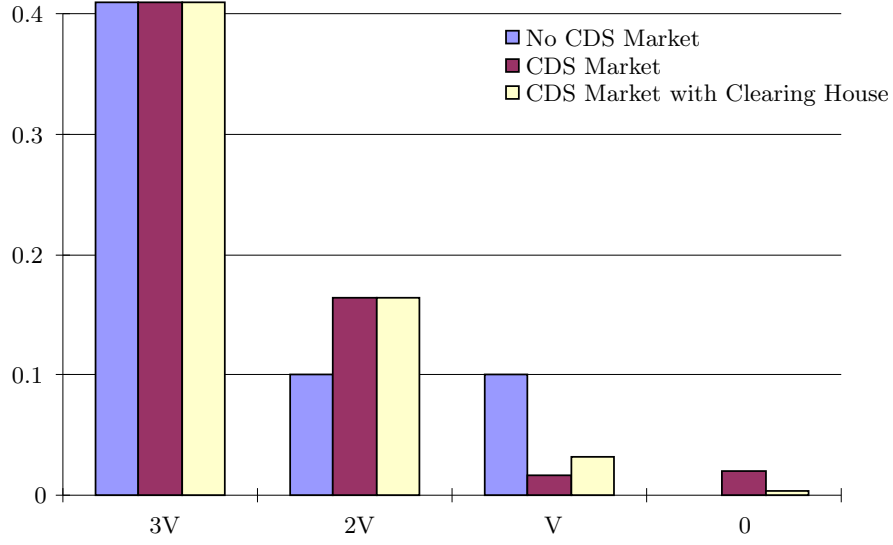


Figure 9: Probability distribution of output at  $t = 2$  with CDS market and central clearing house

According to Corollary 3.3, a CDS market with a central clearing house is always more beneficial than a CDS market without a central clearing house, as shown in Figure 10. In addition, a CDS market with a central clearing house will be optimal given that the social planner is not too risk-averse, or  $f(2V) > f(V)(1 + \frac{\lambda^2}{(1-\lambda)^2})$ . Note that Assumption 3.3 implies that  $\lambda < \frac{1}{2}$ , so we have  $(1 + \frac{\lambda^2}{(1-\lambda)^2}) < 2$ .

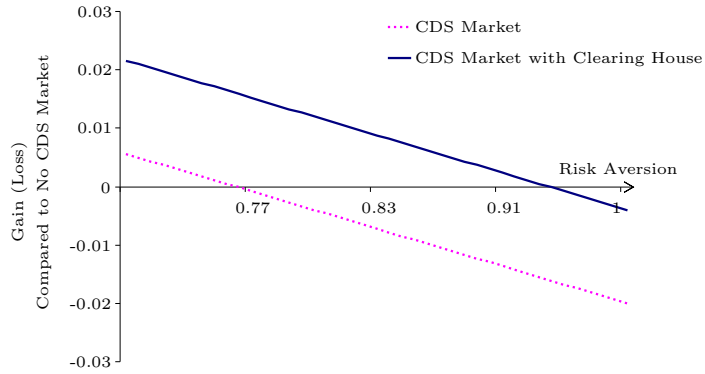


Figure 10: Gain (Loss) of CDS Market with Central Clearing House

I should however point out that there are important aspects of the central clearing house not considered in this paper, such as multi-lateral netting. Moreover, the capital held by the



clearing house may be optimally deployed elsewhere to finance investment projects. Therefore this is by no means a conclusive statement on the social efficiency of clearing houses. Instead, the focus of this paper is on the limitation of clearing houses, as discussed below.

We can see that the systemic risk introduced by the CDS market is not completely eliminated, as the probability that all banks go bankrupt is positive albeit small. This is because the banks still need to take on the default risk of the reference entity when selling CDS contracts, given that a central clearing house only protects traders from counter-party risk. That is, when risk is shared among a small number of banks, it would become more likely that the banks default at the same time, even in the absence of counter-party risk. Hence a central clearing house is not a panacea solution to systemic risk.

Moreover, as shown in the following section, excessive risk taken by one reckless institution could spread to the financial system via the CDS market. A central clearing house under current arrangements cannot prevent such risk-taking behaviour.

### 3.3 Reckless Firm

A firm becomes reckless if it is willing to take excessive risk. To be specific, I assume that the long-term income of the firm becomes 0 with probability  $\gamma$ . This is privately known to the firm at  $t = 0$ .

If the firm does not have any long-term income, it would be willing to take any amount of risk (given a positive expected gain), since it is protected by limited liability. Thus we call such a firm reckless. This assumption can be interpreted in many ways, including poor risk-management, moral hazard or irresponsible traders.

In addition, CDS contracts are private information of the parties involved and not observable by other institutions. In reality, due to a lack of regulation and disclosure requirements, trading information in the CDS market is often not available to third-parties and regulators, especially for bespoke contracts<sup>29</sup>. The information asymmetry in the CDS market is essential

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<sup>29</sup>See Dickinson (2008).

to the results in this section. Although trade information may be accessed through the central clearing house, this will not include the contracts between the banks and the firm, which will be explained below.

Finally, I assume that only CDS contracts between the banks can be cleared. In reality, only contracts between members can be cleared through the central clearing house<sup>30</sup>. Clearing houses have stringent membership requirements and are open to large qualified financial institutions only. Currently, most clearing house members are CDS dealers consisting of large investment banks, as other parties either do not qualify or have no incentives to become a member. Recent efforts by the regulators for CDS clearing were also constrained to inter-dealer contracts. There has been proposals for buy-side clearing, but were either not implemented or not very successful<sup>31</sup>. Furthermore, only standardized contracts can be cleared through the central clearing house. Therefore, there is a significant proportion of CDS contracts not cleared through the clearing house, including contracts with non-dealers and non-standard contracts. Hence I assume in the model that the clearing house can only clear CDS contracts between banks, which implies that the market between banks and the firm would remain opaque.

The presence of the reckless firm would significantly increase systemic risk. The intuition is summarized as follows. When the firm is not reckless, it borrows from bank A who hedges with bank B via the CDS market as in the baseline case. However, when the firm is reckless, in addition to lending, bank A has incentives to contract with the firm. This is because bank A can induce the reckless firm to take on excessive risk (by buying CDS from the firm) and profit from the CDS protection it bought on the firm from bank B, who cannot infer whether the firm is reckless or not. In such a way, the excessive risk taken by the reckless firm spreads to other banks via the CDS market.

Some uncertain event is needed in order for the firm to take such risk. The firm would not sell CDS protection on bank A or bank B, because bank A can only make a profit on the CDS contract between bank A and bank B if both of them remain solvent. Therefore, as

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<sup>30</sup>For an explanation of the structure of clearing houses, see Reuters (2010*b*).

<sup>31</sup>See FOX Business (2010).

shown later, the reckless firm would sell CDS contracts on the liquidity shock, which is the only source of risk available in the model. That is, notional payments of the CDS contracts are triggered when the investors receive a liquidity shock at  $t = 1$ . We can think of those as CDS contracts written on mortgaged-backed securities and other debt instruments that tend to default in a liquidity crisis. This assumption is for the purpose of simplicity only, and introducing additional sources of risk into the model will not change the qualitative results.

### 3.3.1 No CDS Market

Recall that the firm becomes reckless if its long-term payoff is 0 with probability  $\gamma$ . If there is no CDS market, the equilibrium will be the same as in the baseline case, except that the probability distribution of total output would be different, since the reckless firm does not produce any output at  $t = 2$ .

**Proposition 3.4.** *If there is no CDS market, the following is a Nash equilibrium given Assumption 3.1 and 3.2. At  $t = 0$ :*

*Bank A issues debt with price  $2I$  and notional amount  $I + \frac{I - \frac{q}{2}(1-\lambda)r}{1 - \frac{q}{2}}$ , invests in its own project and lends  $I$  to the firm. The notional value of the loan is  $N^F = \frac{I + \frac{q}{2}V}{1 - q}$ .*

*Bank B issues debt with price  $I$  and notional amount  $I$  and invests in its own project*

*The probability distribution of terminal output with no CDS market is*

$$\mathbf{P}(Y = 3V | \text{No CDS}) = (1 - \gamma)(1 - q)$$

$$\mathbf{P}(Y = 2V | \text{No CDS}) = \frac{q}{2} + (1 - q)\gamma$$

$$\mathbf{P}(Y = V | \text{No CDS}) = \frac{q}{2}$$

$$\mathbf{P}(Y = 0 | \text{No CDS}) = 0$$

*Proof.* Follows from Appendix A.1. □

As it is already shown that a central clearing house is beneficial, I omit the case without a central clearing house in the following analysis.

### 3.3.2 CDS Market With Central Clearing House

With a CDS market, bank A is able to hedge its exposure with bank B. However, bank A and the firm may also have incentives to collude and exploit the information asymmetry between bank B and themselves. That is, bank A would pay the reckless firm to take a risky bet that will increase the default probability of the firm, so that bank A would profit from its CDS contracts bought from bank B, who cannot observe or infer whether the firm is reckless or not.

Because bank B anticipates that the firm may become reckless and hence more likely to default, it will charge a CDS premium much higher than in the baseline case. If  $\gamma$  is so large that CDS protection on the firm is too costly, bank A will not have incentives to hedge with bank B when the firm is not reckless. Hence we need to assume that  $\lambda$  and  $\gamma$  are small enough, such that the first inequality in Assumption 3.4 is satisfied. In addition, we need to ensure that  $H$  is large enough so that the bank A is able to repay its debt if the firm's project is successful, as in Assumption 3.4.

**Assumption 3.4.** Assume  $\left(\frac{q}{2}(1-2\lambda) - \frac{\gamma}{2}\left(\frac{(1-\lambda)q}{2} + \lambda(1-q)\right)\right)V > \left(\frac{q}{2} + \frac{\gamma}{2}\left(1 - \frac{q}{2}\right)\right)\left(I - \left(1 - \frac{q}{2}\lambda\right)r\right)$  and  $H \geq \frac{I + \frac{\gamma(1-q)}{4}\left(I + \frac{q}{2}\lambda r - r\right) + \lambda\left(q + \frac{\gamma}{2}(1-q)\right)V}{1-q}$ .

Let's first examine the symmetric information case where CDS contracts are publicly observable. Hence the CDS premium always reflects the true riskiness of the firm who would have no incentives to take extra risk. Under symmetric information, the equilibrium is similar to the one described in Proposition 3.3 in the baseline case and is specified below.

**Proposition 3.5.** (*Symmetric Information*) Suppose CDS contracts are publicly observable. Given Assumption 3.1, 3.2 and 3.4, the following is a Nash equilibrium. At  $t = 0$ :

The optimal contract between bank A and bank B is a CDS contract given by

- $\tau_1^B(\bar{s}) = r - \left(I + \frac{q}{2}\lambda r\right)$

- $\tau_2^B(\underline{s}) = \frac{-q\tau_1^B(\bar{s}) + \frac{q}{2}\lambda V}{1-q}$

The CDS contract is cleared through the clearing house with a fee  $z = -q\lambda\tau_1^B(\bar{s})$

Bank A issues debt with price  $2I$  and notional amount  $2I + \frac{q}{2}\lambda r$ , invests in its own project and lends  $I$  to the firm. The notional value of the loan is  $N^F = \frac{I+q\lambda V}{1-q}$

Bank B issues debt with price  $I$  and notional amount  $I$  and invests in its own project

The clearing house issues debt with price  $-\tau_1^B(\bar{s})$  and notional amount  $N^C = -\tau_1^B(\bar{s})(1+q\lambda)$

The probability distribution of terminal output is

$$\mathbf{P}(Y = 3V | \text{CDS} + \text{Clearing House}) = (1 - \gamma)(1 - q)$$

$$\mathbf{P}(Y = 2V | \text{CDS} + \text{Clearing House}) = \frac{q}{2}(1 + (1 - \lambda)^2) + (1 - q)\gamma$$

$$\mathbf{P}(Y = V | \text{CDS} + \text{Clearing House}) = q\lambda(1 - \lambda)$$

$$\mathbf{P}(Y = 0 | \text{CDS} + \text{Clearing House}) = \frac{q}{2}\lambda^2$$

*Proof.* Follows from Proposition 3.3. □

Now suppose CDS contracts are not publicly observable. Suppose the menu of contracts between bank A and the firm is given by a payment  $\tau_i^F$  ( $i = 1, 2$ ) from bank A to the firm at  $t = 1, 2$  respectively (negative values indicate payments in the opposite directions), conditional on all contractible information. Assume bank A and the firm have equal bargaining powers when contracting with each other and bargain efficiently so that the total surplus is maximized. Note that bank A is not competitive in this case, because bank A can infer whether the firm is reckless or not when contracting with the firm, and hence the firm cannot approach bank B instead. However, bank A is still competitive when lending to the firm in the first place.

Given that bank A has hedged its exposure with bank B and the firm is reckless, bank A

and the firm choose  $\tau_i^F$  conditional on  $s$  and  $l$  to maximize<sup>32</sup>

$$\begin{aligned} & (1-q) \left( \frac{1}{2} (\mathbf{I}(N^F - H \leq \tau_1^F(l) \leq 0)(-\tau_1^F(l) - \tau_2^F(l)) + \mathbf{I}(\tau_1^F(l) < N^F - H)(H - N^F + \tau_1^B(\bar{S}))) \right. \\ & + \frac{1}{2} (\mathbf{I}(N^F - H \leq \tau_1^F(\bar{l}) \leq 0)(-\tau_1^F(\bar{l}) - \tau_2^F(\bar{l})) + \mathbf{I}(\tau_1^F(\bar{l}) < N^F - H)(H - N^F + \tau_1^B(\bar{S}))) \Big) \\ & + (1-q) \left( \frac{1}{2} (\mathbf{I}(N^F - H \leq \tau_1^F(l) \leq 0)(\tau_1^F(l) + \tau_2^F(l)) - \mathbf{I}(\tau_1^F(l) < N^F - H)(H - N^F)) \right. \\ & + \frac{1}{2} (\mathbf{I}(N^F - H \leq \tau_1^F(\bar{l}) \leq 0)(\tau_1^F(\bar{l}) + \tau_2^F(\bar{l})) - \mathbf{I}(\tau_1^F(\bar{l}) < N^F - H)(H - N^F)) \Big) \end{aligned}$$

subject to the individual rationality constraints of bank A

$$\begin{aligned} & (1-q) \left( \frac{1}{2} (\mathbf{I}(N^F - H \leq \tau_1^F(l) \leq 0)(-\tau_1^F(l) - \tau_2^F(l)) + \mathbf{I}(\tau_1^F(l) < N^F - H)(H - N^F + \tau_1^B(\bar{S}))) \right. \\ & + \frac{1}{2} (\mathbf{I}(N^F - H \leq \tau_1^F(\bar{l}) \leq 0)(-\tau_1^F(\bar{l}) - \tau_2^F(\bar{l})) + \mathbf{I}(\tau_1^F(\bar{l}) < N^F - H)(H - N^F + \tau_1^B(\bar{S}))) \Big) \geq 0 \end{aligned}$$

and the firm

$$\begin{aligned} & (1-q) \left( \frac{1}{2} (\mathbf{I}(N^F - H \leq \tau_1^F(l) \leq 0)(\tau_1^F(l) + \tau_2^F(l)) - \mathbf{I}(\tau_1^F(l) < N^F - H)(H - N^F)) \right. \\ & + \frac{1}{2} (\mathbf{I}(N^F - H \leq \tau_1^F(\bar{l}) \leq 0)(\tau_1^F(\bar{l}) + \tau_2^F(\bar{l})) - \mathbf{I}(\tau_1^F(\bar{l}) < N^F - H)(H - N^F)) \Big) \geq 0 \end{aligned}$$

**Proposition 3.6.** (*Asymmetric Information*) Suppose CDS contracts are not publicly observable. Given Assumption 3.1, 3.2 and 3.4, the following is a Nash equilibrium. At  $t = 0$ :

The optimal contract between bank A and bank B is a CDS contract given by

- $\tau_1^B(\bar{s}) = r - (I + \frac{q}{2}\lambda r)$
- $\tau_2^B(\underline{s}) = \frac{-(q + \frac{\gamma(1-q)}{2})\tau_1^B(\bar{s}) + \frac{\lambda}{2}(q + \gamma(1-q))V}{(1-q)(1 - \frac{\gamma}{2})}$

The CDS contract is cleared through the clearing house with a fee  $z = -\lambda(q + \gamma(1-q))\tau_1^B(\bar{s})$

If the firm is reckless, the optimal contract between bank A and the firm is a CDS contract given by

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<sup>32</sup>Here I assume that  $-V \leq \tau_1^F \leq 0$  and  $-V \leq \tau_2^F \leq V$  in order to avoid unnecessary complications in the expression, see the proof of Proposition 3.6.

- $\tau_1^F(\bar{l}) < \frac{I + \frac{\gamma(1-q)}{4} \left( I + \frac{q}{2} \lambda r - r \right) + \lambda \left( q + \frac{\gamma}{2} (1-q) \right) V}{1-q} - H$
- $\tau_2^F(\underline{l}) = H - N^F - \frac{1}{2} \tau_1^B(\bar{s})$

Bank A issues debt with price  $2I$  and notional amount  $2I + \frac{q}{2} \lambda r$ , invests in its own project and lends  $I$  to the firm. The notional value of the loan is

$$N^F = \frac{I + \frac{\gamma(1-q)}{4} \left( I + \frac{q}{2} \lambda r - r \right) + \lambda \left( q + \frac{\gamma}{2} (1-q) \right) V}{1-q}$$

Bank B issues debt with price  $I$  and notional amount  $I$  and invests in its own project

The clearing house issues debt with price  $-\tau_1^B(\bar{s})$  and notional amount

$$N^C = -\tau_1^B(\bar{s}) \left( 1 + \lambda(q + \gamma(1-q)) \right)$$

*Proof.* See Appendix A.4. □

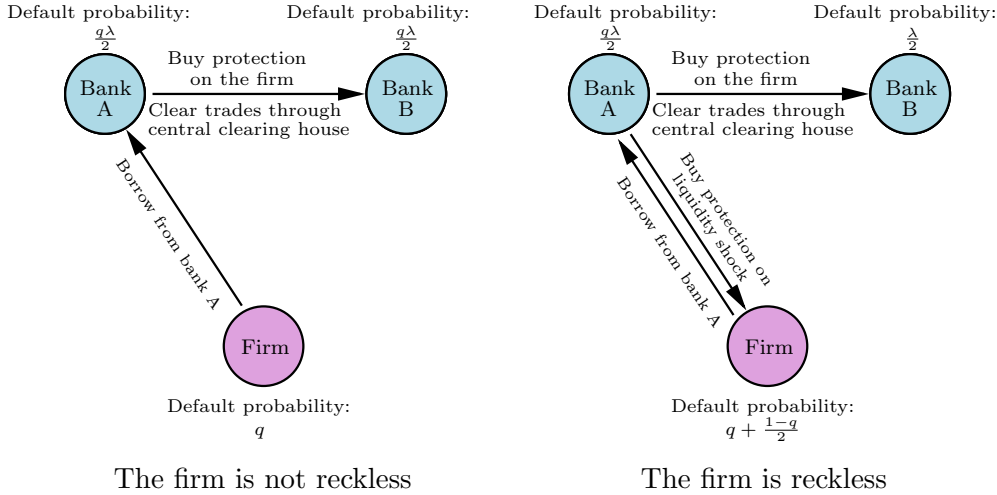


Figure 11: Equilibrium with CDS market and central clearing house

The equilibrium is shown in Figure 11. As specified in Proposition 3.6, Bank A will lend to the firm and hedge with bank B as usual. However, when the firm is reckless, the firm will sell CDS protection on the liquidity shock to bank A. The notional amount of the CDS contract is large enough that the firm would default upon arrival of the liquidity shock. The premium of CDS contracts sold by the firm is such that the surplus is equally split between bank A and the firm.

Let's suppose the firm is reckless and approached bank A. Bank A would immediately infer that the firm is reckless, since otherwise the firm would not have incentives to take risk. By buying CDS protection on the liquidity shock, bank A would increase the default probability of the firm, whose CDS would then become under-priced. Thus bank A has incentives to buy protection on the firm from bank B in order to profit from the mis-pricing. The reckless firm is willing to sell protection as long as bank A promises a high enough premium. Since bank B cannot infer whether the firm is reckless or not, it will trade with bank A, albeit at a higher premium. Also note that it is necessary that only CDS contracts between banks are cleared, since otherwise the clearing house will become informed and reveals the information in the clearing fee<sup>33</sup>.

When the firm is not reckless, bank A will hedge with bank B as in the baseline case. Bank A will not collude with the firm who cares about its long-term payoff  $V$  and will not take excessive risk. Bank A's hedging incentive is still large enough that it is willing to hedge despite a higher premium required by bank B. Note that the firm is worse off ex-ante, but cannot commit not to contract with bank A given the opaqueness of the CDS market.

When the firm becomes reckless, its default probability will increase from  $q$  to  $q + \frac{1-q}{2}$  due to CDS sold on the liquidity shock. Hence the default probability of bank B will increase to  $\frac{\lambda}{2}$ , since the firm will default upon the liquidity shock and bank B will default if and only if it has low income and is unable to refinance. This implies that the default risk of bank B is amplified through the excessive risk taken by the reckless firm.

The information asymmetry in CDS market plays a key role. CDS contracts between bank A and the firm are not observable by other parties, including the central clearing house who only clears CDS contracts between banks, hence CDS protection sold by bank B cannot be conditional on the CDS position of the firm. Moreover, the short-term income of bank A and the firm are not verifiable by other parties in absence of any credit events, as assumed before. Hence bank B cannot observe or infer whether the firm has taken excessive risk. It is also easy

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<sup>33</sup>In reality, when the clearing fee is not necessarily informative, information may still be revealed to clearing house members and the regulators via other means.



to imagine that in reality it would be difficult to write a contract conditional on CDS contracts with a third party, given the legitimate business interest of the third party and the wide range of risky assets available.

**Corollary 3.4.** *If there is a CDS market and a central clearing house at  $t = 0$ , we have<sup>34</sup>*

$$\begin{aligned}\mathbf{P}(Y = 3V|CDS + AI) &= (1 - \gamma)(1 - q) = \mathbf{P}(Y = 3V|CDS + SI) \\ \mathbf{P}(Y = 2V|CDS + AI) &= \frac{q}{2}(1 + (1 - \lambda)^2) + (1 - q)\gamma(1 - \frac{\lambda}{2}) < \mathbf{P}(Y = 2V|CDS + SI) \\ \mathbf{P}(Y = V|CDS + AI) &= q\lambda(1 - \lambda) + (1 - q)\gamma\frac{\lambda}{2} > \mathbf{P}(Y = V|CDS + SI) \\ \mathbf{P}(Y = 0|CDS + AI) &= \frac{q}{2}\lambda^2 = \mathbf{P}(Y = 0|CDS + SI)\end{aligned}$$

and  $\mathbf{E}(f|CDS + AI) < \mathbf{E}(f|CDS + SI)$ .

Moreover, we have  $\mathbf{E}(f|CDS + AI) < \mathbf{E}(f|No\ CDS)$  if and only if

$$f(2V) < f(V)\frac{(1-\lambda)^2+\lambda^2-k}{(1-\lambda)^2-k}, \text{ where } k = \frac{(1-q)\gamma\lambda}{q}.$$

*Proof.* Follows from Proposition 3.5 and Proposition 3.6. □

Figure 12 is the probability distribution of output at  $t = 2$ . We can see that the probability of joint failure of financial institutions ( $\mathbf{P}(Y = V)$ ) is higher under asymmetric information, indicating greater systemic risk.

Figure 13 shows that the benefits of a CDS market is significantly reduced under asymmetric information, as defaults of financial institutions become more correlated due to the excessive risk taken by the firm. This effect can be measured by  $k = \frac{(1-q)\gamma\lambda}{q}$ , which is increasing in  $\lambda$  and  $\gamma$ . The larger  $k$  is, the greater the increase in systemic risk and the less attractive the CDS market becomes.

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<sup>34</sup>SI denotes Symmetric Information and AI denotes Asymmetric Information.

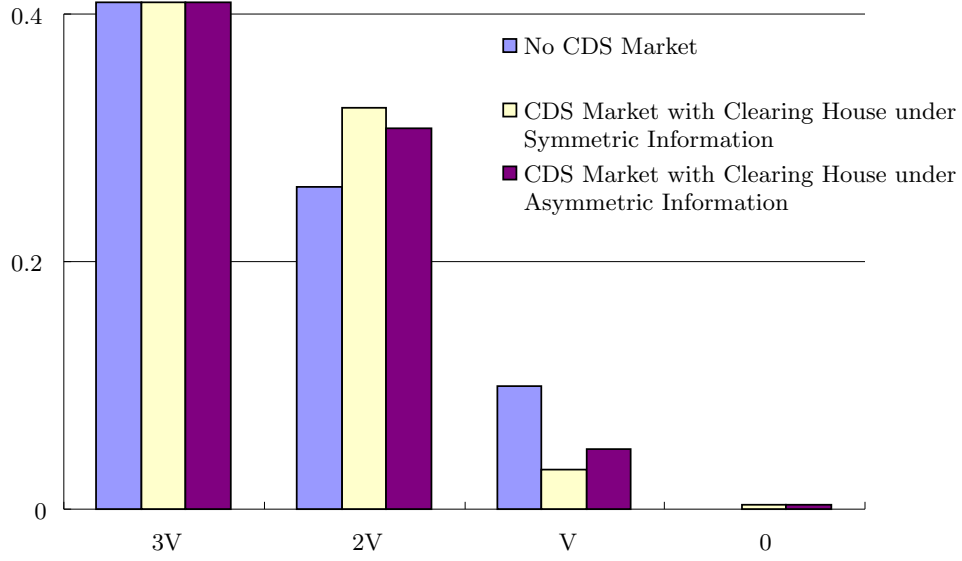


Figure 12: Probability distribution of output at  $t = 2$  with CDS market and central clearing house

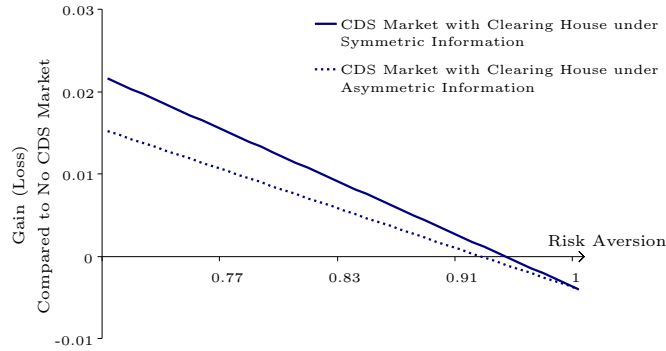


Figure 13: Gain (Loss) of CDS Market

Note that the CDS market plays two roles in this model. First, the reckless firm takes excessive risk by selling CDS protection on risky assets; second, the informed bank (bank A) profits from its private information by buying CDS protection on the firm. Nonetheless, the increase in systemic risk is purely due to the latter. While the reckless firm itself also becomes more likely to default, this will not affect the total output, since the firm will not have any output at  $t = 2$  if it is reckless. This shows that CDS may be dangerous not only for providing a tool for speculation, but also for the propagation of excessive risk.

Such risk-taking behaviour may also be possible with other risky assets, for example, one informed bank could make profits by selling risky assets to a reckless firm while shorting the equity of the reckless firm. However, this is unlikely to be feasible in practice for the following reasons. The informed bank may not have enough risky assets; the reckless firm may not find enough resources to make the purchase; the informed bank may find it difficult to short the equity; the trade may be subject to disclosure requirements or have price impacts. CDSs have a number of features that makes everything easier: it requires no payments upfront; the notional amount of the contract is unlimited; the market is OTC and opaque; the seller will incur large losses with small probabilities, which is perfect for short-term speculation. Nonetheless, it should be pointed out that other OTC derivatives may also be suitable tools to take excessive risk. Therefore the policy implications discussed in the following section apply not only to CDSs but also to other OTC derivatives.

## 4 Policy Implication

As I have shown above, risk-sharing via CDS may contribute to systemic risk in a market with concentrated dealers. Moreover, information asymmetry in the CDS market causes the excessive risk taken by one reckless institution to spread to the entire financial system. Because currently only standard CDS contracts between dealers can be cleared, such information asymmetry remains in presence of a central clearing house. Therefore centralized clearing, albeit beneficial, is by no means a panacea for financial stability<sup>35</sup>.

This implies that policies that mitigate the information asymmetry faced by the regulators and improve supervision of financial institutions, so that any excessive risk-taking behaviour can be promptly identified, would be desirable. Possibilities include mandatory reporting requirements, regulatory access to trade information through repositories<sup>36</sup>, and centralized clearing of all CDS and other OTC derivative contracts.

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<sup>35</sup>As Duffie (2010) pointed out, “*clearing would not have prevented the AIG fiasco*”.

<sup>36</sup>Information on CDS trades registered in DTCC was not directly accessible by the regulators until recently. See Reuters (2010a).

However, some of the policies may have additional costs not considered in the model but are nonetheless important in practice. For example, many argue that centralized clearing and standardization of all CDS contracts would impose burdens on buy-side institutions<sup>37</sup> and curb financial innovation<sup>38</sup>. Hence it is worth pointing out that it may be sufficient to apply such policies only to contracts traded between systemically important institutions<sup>39</sup>. Given a workable definition of systemically important institutions<sup>40</sup>, this could reduce the systemic risk previously embedded in the CDS market while minimizing associated costs.

All the above have focused on limiting the extent to which risk can spread via the CDS market. In addition, it may also be beneficial to reduce the negative consequences associated with such risk. The major OTC derivatives dealers are typically owned by large global financial groups, which also have significant activities in commercial and investment banking<sup>41</sup>. The banking system would become more resilient to the risk in the CDS market if the dealers are less concentrated and less systemically important. Certainly, concentration and engagement in CDS dealing activities of global banks may bring benefits that are outside the scope of this paper but should be taken into account when making policy decisions.

Finally, an important issue not addressed in this paper is the management of risk at the central clearing house. Since central clearing houses concentrate credit and operational risks in itself, a potential clearing house failure could mean disaster for the financial system<sup>42</sup>. Therefore it is very important that the clearing houses be resilient amidst financial market turbulences, with the discipline not to compromise margin and collateral requirements in the face of competitive pressures.

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<sup>37</sup>See Financial Times (2010).

<sup>38</sup>See Stulz (2009).

<sup>39</sup>Systemically important institutions can be conceptually defined as those whose failure would have economically significant spill-over effects which, if left unchecked, could destabilize the financial system and have a negative impact on the real economy. See Thomson (2009).

<sup>40</sup>See IMF (2009) for a practical framework.

<sup>41</sup>See Duffie (2010).

<sup>42</sup>See IMF (2010).

## 5 The Case of AIG

This paper is highly relevant to the events in the recent financial crisis. During 2008, Lehman Brothers suffered severe losses on its exposure to sub-prime mortgage securities, and AIG revealed a huge amount CDS protection sold on CDOs and other debt instruments. Lehman Brothers filed for bankruptcy on 15 September, 2008, and AIG was bailed out subsequently by the U.S. government on 16 September, 2008. Why did the U.S. government decide to abandon Lehman Brothers but rescue AIG?

One of the possible explanations is that the U.S. government decided to rescue AIG not because AIG was the counter-party to many other financial institutions, but because many financial institutions have sold CDS protection on AIG. Out of the \$ 62.1 billion CDS payouts, AIG paid over a half to just two investment banks: Goldman Sachs and Société Générale. Goldman Sachs has revealed during the crisis that it did not have “material exposure” to AIG, implying that it has bought a significant amount of CDS protection on AIG’s debt. Actually, by trading with AIG, Goldman and other counter-parties may have realized the excessive risk AIG was taking and hence had the incentive to actively buy CDS protection on AIG. That is, AIG’s counter-parties possessed private information about the default risk of AIG and could potentially profit from buying protection on AIG from other financial institutions who were unaware of AIG’s exposure.

*“A.I.G.’s trading partners were not innocent victims here, They were sophisticated investors who took enormous, irresponsible risks.”*

- Senator Christopher J. Dodd

AIG Financial Products headed by Joseph Cassano in London had entered into credit default swaps to insure \$441 billion worth of securities. Selling the swaps was “an act of incredible corporate irresponsibility” according to an ex-AIG Financial Products executive<sup>43</sup>. The US government may have realized that a large amount of CDS protection had been written on AIG’s debt and did not want to see the CDS payouts triggered by AIG’s default, hence the decision to bail out the insurance company.

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<sup>43</sup>See Kohlhagen (2010).

*“...we had no idea how much in swaps had been written on AIG itself or by whom. That meant we did not know what the broader effect of an AIG bankruptcy would be.”*

- Eric Dinallo from NY State Insurance Department

Moreover, the Federal Reserve was at first unwilling to disclose the list of CDS counterparties to AIG (although eventually did so under pressure) and then decided to keep the details of AIG's CDS trades secret until 2018. This highlights the extent of information asymmetry in the CDS market and the potential harm to the reputation of the financial industry that may be caused by full disclosure.

*“I would be very concerned that if we gave out the names of counterparties here, people wouldn't want to be doing business with AIG.”*

- Federal Reserve Vice Chairman Donald Kohn

## 6 Conclusion

This paper shows that the CDS market can contribute to systemic risk through not only counter-party risk but also sharing of default risk, given the current level of dealer concentration in the CDS market. Hence, a central clearing house will be effective at eliminating counter-party risk, but is by no means a panacea. Moreover, excessive risk taken by one institution can spread to the entire financial system via the CDS market and pose significant threat to financial stability.

Although this paper has focused on systemic risk with only two banks, its results can be easily extended to models with arbitrarily many banks. When large dealer banks insure each other via the CDS market, excessive risks brought to the financial system may have a multiplier effect. That is, as the default risk of one institution increases, other banks that have insured this institution would also become more risky, and so on. This would have significant implications on gauging the systemic importance of financial institutions.

In addition, an implication of the results of this paper is that, ceteris paribus, adverse information about a bank as a result of irresponsible dealing or mismanagement of risk should

have greater contagious effects on the financial system than other types of bad news. This is due to the potentially large amount of CDS insurance bought on the bank. Hence it would be very interesting to test this hypothesis empirically.

Finally, the risk-aversion of the social planner was used in this paper as a proxy for the systemic externalities of bank failures, i.e. a higher risk-aversion implies lower systemic risk tolerance, and a CDS market is only socially optimal given a low enough risk-aversion. Moreover, there could be other costs associated with efforts to reduce systemic risk, e.g. regulatory expenses, inefficient capital allocation and impediments to financial innovation. Therefore empirical studies measuring systemic externalities<sup>44</sup> and the socially optimal level of systemic risk taking into account all benefits and costs would make significant contributions to policy-making.

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<sup>44</sup>See, for example, Kupiec and Ramirez (2009).

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## A Appendix

### A.1 Proof of Proposition 3.1

*Proof.* First of all, bank A and bank B clearly have incentives to issue debt of  $I$  and invest in the projects which have non-negative expected returns. The debt will be riskless and have notional amount equal to  $I$  given competitive investors.

Now suppose that bank A has issued extra debt  $N^A$  and lent  $I$  to the firm. Given Assumption 3.1, bank A will default if the firm's project fails at  $t = 1$  with probability  $q$ . Since the debt holders are competitive, we have  $N^A = \frac{I - \frac{q}{2}(1-\lambda)r}{1 - \frac{q}{2}}$ .

The payoff (shareholder value) of the firm is 0 if it does not borrow, and is  $(1 - q)(H - N^F + V)$  if it does. The payoff of bank A is  $(1 - \lambda)r + V$  if it does not lend, and is  $(1 - q)((1 - \lambda)r + N^F - N^A + V) + \frac{q}{2}((1 - \lambda)r - N^A + V)$  if it does. Since bank A is competitive, its participation constraint is binding, which implies  $N^F = \frac{I + \frac{q}{2}V}{1 - q}$ . Given Assumption 3.2, we have  $H \geq N^F > N^A$ , which ensures that bank A survives if the firm's project is successful. Finally, the individual rationality constraint of the firm is clearly satisfied given limited liability. This completes the proof. □

### A.2 Proof of Proposition 3.2

*Proof.* First of all, bank A and bank B clearly have incentives to issue debt and invest in the projects which have non-negative expected returns. Moreover, bank A will raise extra debt and lend to the firm as shown in the proof of Proposition 3.1.

Let's examine the maximization problem of bank A when contracting with bank B. Note that a payment from bank A to bank B ( $\tau_1^B > 0$ ) at  $t = 1$  cannot be Pareto improving, since bank A would subject to a higher default risk. We also disregard the cases where  $\tau_i^B > V$  or  $\tau_i^B < -V$  as institutions are protected by limited liability. So let's assume  $-V \leq \tau_1^B \leq 0$  and  $-V \leq \tau_i^B \leq V$ . The participation constraint binds since bank B is competitive. Substituting

the binding participation constraint, bank A maximizes

$$\begin{aligned}
& (1 - \frac{q}{2})((1 - \lambda)r + V) + (1 - q)(N^F - N^A) + \frac{1 - q}{2} \left( \mathbf{I}(0 > \tau_1^B(\underline{s}, \bar{l}) \geq -r) \lambda(r - \tau_2^B(\underline{s}, \bar{l}) - V) \right. \\
& + \mathbf{I}(\tau_1^B(\underline{s}, \bar{l}) < -r)(-\tau_2^B(\underline{s}, \bar{l}) - V) \Big) + \frac{q}{2} \left( \mathbf{I}(0 > \tau_1^B(\bar{s}, \bar{l}) > r - N^A) ((1 - \lambda)^2(\tau_1^B(\bar{s}, \bar{l}) + \tau_2^B(\bar{s}, \bar{l})) \right. \\
& + \lambda(1 - \lambda)\tau_1^B(\bar{s}, \bar{l}) - \lambda V) + \mathbf{I}(r - N^A \geq \tau_1^B(\bar{s}, \bar{l}) \geq -r) ((1 - \lambda)^2(\tau_1^B(\bar{s}, \bar{l}) + \tau_2^B(\bar{s}, \bar{l})) \\
& + \lambda(1 - \lambda)\tau_1^B(\bar{s}, \bar{l}) - \lambda V + (1 - \lambda)^2(-\tau_1^B(\bar{s}, \bar{l}) - (N^A - r) - \tau_2^B(\bar{s}, \bar{l}) + V)) \\
& \left. + \mathbf{I}(\tau_1^B(\bar{s}, \bar{l}) < -r) (- (1 - \lambda)r - V + (1 - \lambda)^2(-\tau_1^B(\bar{s}, \bar{l}) - (N^A - r) - \tau_2^B(\bar{s}, \bar{l}) + V)) \right)
\end{aligned}$$

It is easy to see that the above expression is maximized when  $\tau_1^B(\underline{s}, \bar{l}) = 0$  and  $r - N^A \geq \tau_1^B(\bar{s}, \bar{l}) \geq -r$  given

$$\begin{aligned}
& (1 - \lambda)^2(\tau_1^B(\bar{s}, \bar{l}) + \tau_2^B(\bar{s}, \bar{l})) + \lambda(1 - \lambda)\tau_1^B(\bar{s}, \bar{l}) - \lambda V \\
& + (1 - \lambda)^2(-\tau_1^B(\bar{s}, \bar{l}) - (N^A - r) - \tau_2^B(\bar{s}, \bar{l}) + V) > 0 \quad (\text{A.1})
\end{aligned}$$

Suppose we have a CDS contract with transfers given by  $\tau_1^B(\bar{s}) = r - N^A$  and  $\tau_2^B(\underline{s}) > 0$ . Substituting into the participation constraint

$$(1 - q)\tau_2^B(\underline{s}) + \frac{q}{2}\tau_1^B(\bar{s}) + \frac{q}{2}((1 - \lambda)\tau_1^B(\bar{s}) - \lambda V) = 0$$

Hence

$$\tau_2^B(\underline{s}) = \frac{-q(1 - \frac{\lambda}{2})\tau_1^B(\bar{s}) + \frac{q}{2}\lambda V}{1 - q}$$

In addition, given the CDS contract, we have  $N^A = \frac{I - \frac{q}{2}\lambda(1 - \lambda)(r - \tau_1^B(\bar{s}))}{1 - q + \frac{q}{2} + \frac{q}{2}(1 - \lambda)^2}$ , or  $N^A = \frac{I}{1 - \frac{q}{2}\lambda}$ .

Moreover, inequality A.1 can be rewritten as

$$(1 - \lambda)\tau_1^B(\bar{s}) - \lambda V + (1 - \lambda)^2 V > 0$$

It can be shown that the inequality above is satisfied given Assumption 3.3. Hence the CDS contract is the optimal contract.

Finally, given that bank A is competitive when lending to the firm, its participation constraint is binding

$$(1-q)((1-\lambda)r + V + N^F - N^A - \tau_2^B(\underline{s})) + \frac{q}{2}((1-\lambda)r + V - \tau_1^B(\bar{s}) - N^A) + \frac{q}{2}(1-\lambda)^2V = (1-\lambda)r + V$$

hence the notional amount of the loan is  $N^F = \frac{I + \frac{q}{2}(1-(1-\lambda)^2 + \lambda)V}{1-q}$ . This completes the proof.  $\square$

### A.3 Proof of Proposition 3.3

*Proof.* First of all, bank A and bank B clearly have incentives to issue debt and invest in the projects which have non-negative expected returns.

Let's look at the decision of the clearing house first. Obviously, clearing is only Pareto-improving if the clearing house is able to make the full payment on the CDS contract being cleared, since otherwise bank A will not be able to survive anyway. Therefore, given that the clearing house and its debt holders are competitive, we have

$$-\frac{1-q}{2} \left( \lambda \mathbf{I}(0 > \tau_1^B(\underline{s}, \bar{l}) \geq -r) \tau_1^B(\underline{s}, \bar{l}) + \mathbf{I}(\tau_1^B(\underline{s}, \bar{l}) \leq -r) \tau_1^B(\underline{s}, \bar{l}) \right) - \frac{q}{2} \left( \lambda \mathbf{I}(0 > \tau_1^B(\bar{s}, \bar{l}) \geq -r) \tau_1^B(\bar{s}, \bar{l}) + \mathbf{I}(\tau_1^B(\bar{s}, \bar{l}) \leq -r) \tau_1^B(\bar{s}, \bar{l}) \right) = \frac{1}{2}z$$

Note that the clearing fee is only paid if there is no liquidity shock, since the fee is exempt if the clearing house becomes bankrupt.

Similar to the proof of Proposition 3.2, bank A maximizes

$$\begin{aligned}
& (1 - \frac{q}{2})((1 - \lambda)r + V) + (1 - q)(N^F - N^A) + \frac{1 - q}{2} \left( \mathbf{I}(0 > \tau_1^B(\underline{s}, \bar{l}) \geq -r) \lambda(r - \tau_2^B(\underline{s}, \bar{l}) - V) \right. \\
& + \mathbf{I}(\tau_1^B(\underline{s}, \bar{l}) < -r)(-\tau_2^B(\underline{s}, \bar{l}) - V) \Big) + \frac{q}{2} \left( \mathbf{I}(0 > \tau_1^B(\bar{s}, \bar{l}) > r - N^A) ((1 - \lambda)^2(\tau_1^B(\bar{s}, \bar{l}) + \tau_2^B(\bar{s}, \bar{l})) \right. \\
& + \lambda(1 - \lambda)\tau_1^B(\bar{s}, \bar{l}) - \lambda V) + \mathbf{I}(r - N^A \geq \tau_1^B(\bar{s}, \bar{l}) \geq -r) ((1 - \lambda)^2(\tau_1^B(\bar{s}, \bar{l}) + \tau_2^B(\bar{s}, \bar{l})) \\
& + \lambda(1 - \lambda)\tau_1^B(\bar{s}, \bar{l}) - \lambda V + (1 - \lambda)(-\tau_1^B(\bar{s}, \bar{l}) - (N^A - r) - \tau_2^B(\bar{s}, \bar{l}) + V)) \\
& + \mathbf{I}(\tau_1^B(\bar{s}, \bar{l}) < -r) (-(1 - \lambda)r - V + (1 - \lambda)(-\tau_1^B(\bar{s}, \bar{l}) - (N^A - r) - \tau_2^B(\bar{s}, \bar{l}) + V)) \Big) \\
& + \frac{1 - q}{2} \left( \lambda \mathbf{I}(0 > \tau_1^B(\underline{s}, \bar{l}) \geq -r) \tau_1^B(\underline{s}, \bar{l}) + \mathbf{I}(\tau_1^B(\underline{s}, \bar{l}) \leq -r) \tau_1^B(\underline{s}, \bar{l}) \right) \\
& + \frac{q}{2} \left( \lambda \mathbf{I}(0 > \tau_1^B(\bar{s}, \bar{l}) \geq -r) \tau_1^B(\bar{s}, \bar{l}) + \mathbf{I}(\tau_1^B(\bar{s}, \bar{l}) \leq -r) \tau_1^B(\bar{s}, \bar{l}) \right)
\end{aligned}$$

It is easy to see that the above expression is maximized when  $\tau_1^B(\underline{s}, \bar{l}) = 0$  and  $r - N^A > \tau_1^B(\bar{s}, \bar{l}) \geq -r$  given

$$\begin{aligned}
& (1 - \lambda)\tau_1^B(\bar{s}, \bar{l}) + (1 - \lambda)^2\tau_2^B(\bar{s}, \bar{l}) - \lambda V \\
& + (1 - \lambda)(-\tau_1^B(\bar{s}, \bar{l}) - (N^A - r) - \tau_2^B(\bar{s}, \bar{l}) + V) + \lambda\tau_1^B(\bar{s}, \bar{l}) > 0 \quad (\text{A.2})
\end{aligned}$$

Suppose we have a CDS contract with transfers given by  $\tau_1^B(\bar{s}) = r - N^A$  and  $\tau_2^B(\underline{s}) > 0$ . The participation constraint of bank B becomes

$$(1 - q)\tau_2^B(\underline{s}) + \frac{q}{2}\tau_1^B(\bar{s}) + \frac{q}{2}((1 - \lambda)\tau_1^B(\bar{s}) - \lambda V) - \frac{1}{2}z = 0$$

where

$$z = -\lambda q \tau_1^B(\bar{s})$$

Hence

$$\tau_2^B(\underline{s}) = \frac{-q\tau_1^B(\bar{s}) + \frac{q}{2}\lambda V}{1 - q}$$

In addition, given the CDS contract, we have  $N^A = \frac{I + \frac{q}{2}\lambda\tau_1^B(\bar{s})}{1 - q + \frac{q}{2} + \frac{q}{2}(1 - \lambda)}$ , or  $N^A = I + \frac{q}{2}\lambda r$ . Moreover,

inequality A.2 can be rewritten as

$$\tau_1^B(\bar{s}) + (1 - 2\lambda)V > 0$$

It can be shown that the inequality above is satisfied given Assumption 3.3. Hence the CDS contract is the optimal contract.

Finally, given that bank A is competitive when lending to the firm, the notional amount of the loan is  $N^F = \frac{I+q\lambda V}{1-q}$ . Moreover, given that the clearing house is competitive, we must have  $N^C = -\tau_1^B(\bar{s}) + z$ , where  $N^C$  denotes the notional amount of debt issued by the clearing house. This completes the proof.  $\square$

#### A.4 Proof of Proposition 3.6

*Proof.* First of all, bank A and bank B clearly have incentives to issue debt and invest in the projects which have non-negative expected returns.

Suppose the firm is not reckless. The equilibrium will be the same as in Proposition 3.3, except that the CDS premium paid by bank A will be different. Specifically, bank A's objective function is maximized when  $\tau_1^B(\underline{s}, \bar{l}) = 0$  and  $r - N^A > \tau_1^B(\bar{s}, \bar{l}) \geq -r$  given

$$(1 - \lambda)\tau_1^B(\bar{s}, \bar{l}) + (1 - \lambda)^2\tau_2^B(\bar{s}, \bar{l}) - \lambda V + (1 - \lambda)(-\tau_1^B(\bar{s}, \bar{l}) - (N^A - r) - \tau_2^B(\bar{s}, \bar{l}) + V) \\ + \lambda\tau_1^B(\bar{s}, \bar{l}) + \frac{1-q}{2}\left(\gamma(1 - \lambda)\tau_1^B(\bar{s}, \bar{l}) - \gamma\lambda V - \gamma\tau_2^B(\underline{s}, \bar{l})\right) + \frac{q}{2}\gamma\lambda\tau_1^B(\bar{s}, \bar{l}) > 0 \quad (\text{A.3})$$

Suppose we have a CDS contract with transfers given by  $\tau_1^B(\bar{s}) = r - N^A$  and  $\tau_2^B(\underline{s}) > 0$ . Then the participation constraint of bank B binds

$$(1 - q)\tau_2^B(\underline{s}) + \frac{q}{2}\tau_1^B(\bar{s}) + \frac{q}{2}((1 - \lambda)\tau_1^B(\bar{s}) - \lambda V) \\ + \frac{1-q}{2}(\gamma(1 - \lambda)\tau_1^B(\bar{s}) - \gamma\lambda V - \gamma\tau_2^B(\underline{s})) - \frac{1}{2}z = 0$$



where

$$z = -\lambda(q + \gamma(1 - q))\tau_1^B(\bar{s})$$

Hence

$$\tau_2^B(\underline{s}) = \frac{-(q + \frac{\gamma(1-q)}{2})\tau_1^B(\bar{s}) + \frac{\lambda}{2}(q + \gamma(1 - q))V}{(1 - q)(1 - \frac{\gamma}{2})}$$

In addition, given the CDS contract, we have  $N^A = I + \frac{q}{2}\lambda r$ . Moreover, inequality A.3 can be rewritten as

$$\left(\frac{q}{2}(1 - 2\lambda) - \frac{\gamma}{2}\left(\frac{(1 - \lambda)q}{2} + \lambda(1 - q)\right)\right)V > -\left(\frac{q}{2} + \frac{\gamma}{2}(1 - \frac{q}{2})\right)\tau_1^B(\bar{s})$$

It can be shown that the inequality above is satisfied given Assumption 3.4. Hence the CDS contract is the optimal contract.

Now let's suppose the firm is reckless. Clearly bank A still lends to the firm and hedges with bank B in this case. The hedging contract must be identical as above, since otherwise bank B would infer that the firm is reckless. Hence we focus on the contract between bank A and the firm. Note that a payment from bank A to the firm ( $\tau_1^B > 0$ ) at  $t = 1$  cannot be Pareto improving, since bank A would subject to a higher default risk. Hence we assume  $\tau_1^F \leq 0$ . It is easy to see that the total surplus is maximized subject to the individual rationality constraints when  $\tau_1^F(\underline{l}) < N^F - H$  or  $\tau_1^F(\bar{l}) < N^F - H$ , but not both. We focus on the latter as it has more interesting implications.

So suppose we have a CDS contract with transfers given by  $\tau_1^F(\bar{l}) < N^F - H$  and  $\tau_2^F(\underline{l}) > 0$ . The surplus is equally split between bank A and the firm

$$(1 - q)\left(-\frac{1}{2}\tau_2^F(\underline{l}) + \frac{1}{2}(H - N^F - \tau_1^B(\bar{s}))\right) = (1 - q)\left(\frac{1}{2}\tau_2^F(\underline{l}) - \frac{1}{2}(H - N^F)\right)$$

This implies

$$\tau_2^F(\underline{l}) = H - N^F - \frac{\tau_1^B(\bar{s})}{2}$$

Given that bank A is competitive when lending to the firm, we have

$$(1-q)((1-\lambda)r + V + N^F - N^A - (1 - \frac{\gamma}{2}\tau_2^B(\underline{s})) + \frac{q}{2}((1-\lambda)r + V - \tau_1^B(\bar{s}) - N^A) \\ + \frac{q}{2}(1-\lambda)V + \gamma(1-q)(-\frac{1}{2}\tau_1^B(\bar{s}))) = (1-\lambda)r + V$$

hence the notional amount of the loan is  $N^F = \frac{I + \frac{\gamma(1-q)}{4}(I + \frac{q}{2}\lambda r - r) + \lambda(q + \frac{\gamma}{2}(1-q))V}{1-q}$ . In addition, given that the clearing house is competitive, we must have  $N^C = -\tau_1^B(\bar{s}) + z$ , where  $N^C$  denotes the notional amount of debt issued by the clearing house.

Finally, we need to show that bank A does not have incentives to buy CDS from the firm when the firm is not reckless. If bank A buys CDS from the firm, bank A receives  $\tau_1^B(\bar{s})$  with  $\frac{1}{2}$  probability, but the firm loses  $V$  with another  $\frac{1}{2}$  probability. This is clearly not optimal, since  $V > \tau_1^B(\bar{s})$ . This completes the proof.

□

# Investor Cash Flow and Mutual Fund Behavior

Zijun Liu

Zhigang Qiu

## **Abstract**

We study the behavior of mutual fund managers in a multi-period model with interim cash inflows from investors. Given that the investors would only invest in a fund that has the potential to increase in value, the fund manager has incentives to buy overvalued assets in order to mimic a profitable trade and attract new investors. This would lead to higher risky asset prices and a greater-than-optimal proportion of investment in risky assets in the fund management industry.

# 1 Introduction

We study how the interim cash flow from investors can affect the behavior of mutual fund managers. The majority of mutual funds are open-ended, meaning that they raise money by selling shares of the fund to the public. An investor will usually purchase fund shares directly from the fund itself rather than from other shareholders, and the shares of the fund are priced at the net asset value (NAV) of the fund. Similarly, the fund buys back shares priced at NAV when the investors want to sell. In this paper, we focus exclusively on this type of mutual funds.

The fee structure of mutual funds varies, but most funds charge management fees as a fixed fraction of the total asset under management<sup>1</sup>. This should usually encourage the fund manager to act in the interests of the investors, since his payoff is maximized by maximizing the profits of the fund. However, given that the investors can invest in the fund at different times, the manager will actually be interested in maximizing the total size of the fund, which includes both profits and investments from new investors.

We argue that such incentives would lead the fund manager to trade overvalued assets at the expense of the investors. The investors can obtain information about the future profitability of the fund after observing the fund's portfolio holdings<sup>2</sup>. Therefore, the investors have incentives to purchase more shares of the fund, if they believe that the fund's NAV will rise in the future. Anticipating this, the fund manager will be reluctant to hold cash, even when assets are overvalued, because the investors will be unwilling to invest in a fund that does not have the potential to make profits. This would lead the manager to invest in overvalued assets in order to attract new cash flows.

There is ample anecdotal evidence on the fund managers' reluctance to sit on cash, since by having a cash position too big, the fund managers would send a bad signal to the investors

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<sup>1</sup>See Dangl et al. (2011).

<sup>2</sup>The SEC requires all U.S. mutual funds to increase the frequency of public disclosure of security holdings from semiannual to quarterly, effective May 2004. According to the SEC, the purpose of increasing disclosure frequency is "...to provide better information to investors about fund costs, investments, and performance" (see <http://sec.gov/rules/final/33-8393.pdf>).

who “gave them the money to put in stocks”. For example, Investopedia stated that “*When an investment fund has a large cash position, it is often a sign that it sees few attractive investments in the market and is comfortable sitting on the sidelines.*” Moreover, according to Ryan Leggio, an analyst from Morningstar, “*when an investor initially invests in a fund, they probably do expect it to be fully invested*”. Hence, in order to signal that the fund has taken profitable trading opportunities, fund managers have incentives to hold less cash and invest more in risky assets. Indeed, “*the mutual fund industry has moved toward becoming more fully invested in common stocks, as opposed to bonds and cash*”, as documented in Wermers (2000).

We develop a simple multi-period model ( $t = 0, 1, 2$ ) of portfolio delegation with risk-averse market makers to illustrate the above intuition. There is one risky asset which is traded at  $t = 0$  and pays off at  $t = 2$ . The asset’s payoff consists of two components: a fundamental component and a random component. A fund manager can invest in the asset on behalf of the investors who do not have direct access to the market. In addition, there is a noisy demand from liquidity traders. A competitive and risk-averse market maker submits an optimal demand to clear the market at  $t = 0$ .

We assume that both the fundamental component of the asset’s payoff and the noisy demand are observable to the fund manager and the market maker. In the baseline case with no interim cash flows, the fund manager (subject to the short-sale constraint) is only willing to buy the asset when there is a negative demand shock, i.e. when the noisy demand from the liquidity traders is negative and the asset is undervalued. However, this is no longer the case if we introduce interim cash flows from the investors.

Suppose that at  $t = 1$  there will be some new investors who can observe the manager’s portfolio holdings (and hence his trading actions at  $t = 0$ ) and invest in the fund. If the manager did not trade at  $t = 0$ , the new investors will realize that the fund’s NAV will not rise at  $t = 2$ . Because the investors will pay management fees proportional to their interim investments, it is optimal not to invest at all if the manager did not trade at  $t = 0$ , i.e. the fund is not expected to make any profits.

Anticipating the investor’s behavior, the fund manager has incentives to submit a positive

demand even when the asset is not undervalued, in order to pretend that he has made a profitable trade. The interim investors cannot fully infer the information from the asset price and the manager's action, because they cannot observe the fundamental component of the asset's payoff nor the demand shock. Hence they would still choose to invest in the fund given positive expected profits. We find that there exists an equilibrium in which the manager almost always submits a positive demand and the investors invest in the fund.

Therefore we show that, given potential interim cash flows from the investors, mutual fund managers would have incentives to buy overvalued assets, particularly in illiquid and volatile markets where potential mis-pricing is large. This will lead to higher risky asset prices and a higher-than-optimal proportion of investment in risky assets in the mutual fund industry.

The rest of our paper is organized as follows. Section 2 is a brief literature review. The model setup is described in Section 3, and the model equilibrium is analyzed in Section 4. Finally we conclude in Section 5.

## 2 Related Literature

Our paper is closely related to the literature on delegated portfolio management and asymmetric information asset pricing. A large literature focuses on the relationship between the fund's past performance and investor cash flows, e.g. an empirical study by Chevalier and Ellison (1997) found that there exists a non-linear and convex flow-performance relationship. Huang et al. (2007) provided a theoretical framework with Bayesian updating by introducing an investor participation cost and assuming the fund return as a function of the manager's ability. They show that given the participation cost the investors would invest in the fund when the performance of the fund is good but are reluctant to withdraw their money when the performance is bad. Those papers all focus on the asymmetric relationship between the fund performance and the investors' cash flows. In our paper, however, investors base their decisions not on the past performance of the fund but on the portfolio holdings of the fund.

There are other papers in the delegated portfolio management literature that focus on the

asymmetric information about the fund manager's ability. For example, Dasgupta and Prat (2006) show that the uninformed managers will act like noise traders because they care about their future reputation (career concerns). Scotti (2007) extends the Dasgupta and Prat (2006) model in an order-driven market. These two papers focus on the excessive trading of the uninformed fund managers due to their reputation concerns, which can explain high trading volumes in the financial market. Our paper studies excessive trading from a different angle in which the informed fund managers trade to attract future cash flows.

In this sense, one paper that is very close to ours is Dow and Gorton (1997). In this paper, the good manager will trade randomly (churn) without any private information due to the nature of the optimal contract, which rules out bad managers by not rewarding inactivity. This is quite similar to the case in our paper where the fund manager buys the asset without a negative demand shock. Nonetheless, our paper differs from Dow and Gorton (1997) by providing a different incentive of trading without introducing heterogeneous managers.

Our paper is also related to the literature on asset price bubbles. Allen and Gorton (1993) show that a bubble can persist because the uninformed fund managers will churn. Abreu and Brunnermeier (2003) show that bubbles persist because of market-timing incentives and lack of synchronization among arbitrageurs. In our paper, we also show that the fund manager will ride the bubble (buy the overvalued asset), but with a different underlying mechanism.

Moreover, our paper is also related to the literature on limits to arbitrage, which tries to explain that the ability of the arbitrageurs is limited in some way so that they can not take the correct actions to push the asset prices in line with the fundamentals. For example, De Long et al. (1990) show that arbitrageurs fail to take the arbitrage because of the noise trader risk. In their model, arbitrage opportunities are limited because the arbitrageurs care about short term performance and must liquidate their position in a short time horizon. Since the noise traders can affect the price in the short run, the arbitrageurs face the risk that the price may deviate from fundamentals when they liquidate their positions. Shleifer and Vishny (1997) show another type of limits to arbitrage. In their model, the supply of capital is a function of the past performance of the fund. If the performance is poor, investors can withdraw their

money so that the capital is limited when the arbitrageurs need it most. Gromb and Vayanos (2002), among others, modeled capital limitations of arbitrageurs in a segmented market. All those papers, however, show that the fund managers can not take the correct trading action due to exogenous limitations (short horizon, or lack of capital). In our paper, we show that the fund manager fails to arbitrage without assuming any limitations to the fund manager.

### 3 Model Setup

We consider a discrete-time model with 3 dates,  $t = 0, 1, 2$ . There are three classes of agents: fund managers, investors and market maker.

There is one risky asset that pays off at period 2. Its payoff has two components: a fundamental component denoted by  $d$ , and a random component denoted by  $\epsilon$ , such that  $d$  has a continuous uniform distribution on the interval  $[L, H]$ , and  $\epsilon \sim N(0, \sigma^2)$ . Assume that the total supply of the asset is  $s$ . The risk-free rate is normalized to zero for simplicity.

#### 3.1 Fund Managers

There is a fund manager who has no initial wealth. He can trade the risky asset on behalf of the investors, and will be compensated a fixed fraction  $k$  of the terminal value of the fund at period 2. The manager is risk-averse with CARA risk aversion  $\alpha_f$ . Moreover, the manager perfectly observes  $d$  but cannot observe  $\epsilon$ . The fund manager behaves strategically to maximize his terminal wealth. Assume that the fund has only cash, of value  $m$ , at period 0<sup>3</sup>. The fund will be liquidated at period 2. Finally, we assume that the fund manager is subject to the short-sale constraint<sup>4</sup>.

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<sup>3</sup>This represents the initial investment in the fund by outside investors. For simplicity, we do not model the investment decisions of the investors at  $t = 0$  explicitly, although it would be rather trivial to do so.

<sup>4</sup>The short-sale constraint is actually not used in our model, due to our simplifying assumption that the supply of the asset is equal to the positive noisy demand. However, the short-sale constraint would become important should we relax this assumption. Thus we keep the constraint as it is realistic and makes our model more robust.



### 3.2 Trading

The risky asset can only be traded at period 0. There is a competitive risk-averse market maker with CARA risk-aversion  $\alpha_m$ . The market maker is informed about  $d$  but not  $\epsilon$ , hence the market maker and the fund manager have the same information about the asset's payoff.

In addition, at period 0, there will be a noisy demand  $u$  from liquidity traders, which can be  $s$  with probability  $\frac{1}{2}$  and  $-s$  otherwise<sup>5</sup>. The noisy demand is observable to both the market maker and the fund manager, but not to the investors.

### 3.3 Investors

At period 1, there will be an investor (or a group of investors) with wealth  $W$  who cannot directly access the asset market and decides whether to invest in the fund in the form of share purchases. Each share is priced at NAV and entitled to an equal fraction of the fund's terminal value, as described in the introduction. Hence the payoff that the investor receives at  $t = 2$  depends on his initial investment as a proportion of the fund's total NAV at  $t = 1$ . That is, if the investor invested  $n$  in the fund, his payoff would be  $\frac{n(1-k)}{m+n}$  of the terminal value of the fund, after the management fee. As a result, the investor is going to invest in the fund only if the expected return of the fund exceeds  $k$ .

The investor is not informed about either  $d$  or  $\epsilon$ , but he can observe the fund's portfolio (i.e. the manager's trade at period 0). In addition, the investor is risk-neutral<sup>6</sup>.

### 3.4 Time line

The time line of the model is as shown below. At period 0, the fund manager makes trading decisions. At period 1, the new investors decide whether to invest. At period 2, after  $d + \epsilon$

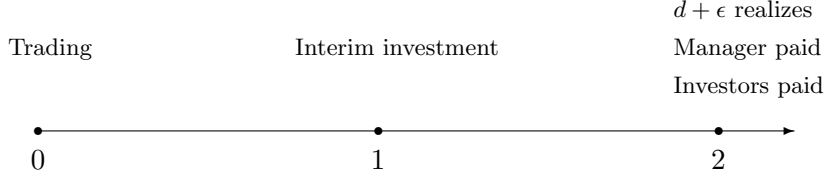
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<sup>5</sup>Assuming that the noisy demand has the same absolute magnitude as the supply is for the purpose of simplicity only.

<sup>6</sup>This assumption greatly simplifies the solution of the model. An equivalent assumption is that there is a continuum of CARA risk-averse investors, each having an arbitrarily small amount of wealth compared to the total fund size.

realizes, the manager gets paid and the investors share the remaining value of the fund.

Figure 1: Time line



## 4 Equilibrium

In order to better illustrate the implication of interim investment, we are going to analyze the equilibrium of the model in two cases. In the baseline case, there is no asymmetric information, i.e. the investors perfectly observe the values of  $u$  and  $d$ . In the interim investment case, investors cannot observe  $u$  and  $d$ . We are going to show that the potential of interim investment as well as asymmetric information may lead to the fund manager trading against his information and riding the bubble at the expense of the investors.

**Assumption 1.**  $\frac{2\alpha_m^2 s^2 \sigma^2}{2\alpha_m + k\alpha_f} < W < \frac{2\alpha_m s \sigma}{2\alpha_m + k\alpha_f} \sqrt{\frac{(1-k)\alpha_f m}{2}} - m$ ,

*This inequality is valid if*

- $\alpha_f > \frac{2(\alpha_m s^2 \sigma + m)^2}{m s^2}$  and
- $k < \frac{s\alpha_m}{m\alpha_f} \left( \sqrt{2\alpha_f m} - 2\alpha_m s \sigma \right) - \frac{2\alpha_m}{\alpha_f}$

This assumption alone can ensure that the proposed equilibrium exists. It may appear complicated and we shall explain its intuition later.

### 4.1 Baseline Case: Symmetric Information

In this section, all the agents have the same information set, i.e. the investors can observe the value of  $d$  and  $u$ . This is the first-best outcome.

**Lemma 1.** *The market clearing price is  $P = d + \alpha_m(x + u - s)\sigma^2$  where  $x$  denotes the demand of the fund manager.*

This is a standard result that follows from risk-averse market makers with CARA utility and normally distributed noise. As we can see, the asset price is decreasing with the total demand from the fund manager and noise traders.

**Proposition 1.** *The optimal demand of the fund manager is*

$$x = \begin{cases} \frac{2\alpha_m s}{2\alpha_m + k\alpha_f} & \text{if } u = -s \\ 0 & \text{if } u = s \end{cases}$$

*the equilibrium price is*

$$P = \begin{cases} d - \frac{2(\alpha_m + k\alpha_f)}{2\alpha_m + k\alpha_f} \alpha_m \sigma^2 s & \text{if } u = -s \\ d & \text{if } u = s \end{cases}$$

*and the investors will invest*

$$n = \begin{cases} W & \text{if } u = -s \\ 0 & \text{if } u = s \end{cases}$$

Because information is symmetric, investors will invest if and only if the asset is undervalued, which is perfectly observable. This is because the fund can only make profits when the asset is undervalued. Given CARA utility and normally distributed noise, the manager's maximization problem is

$$\max \mathbf{E}[kx(d + \epsilon - P)|u, d] - \frac{\alpha_f}{2} \text{Var}[kx(d + \epsilon - P)|u, d] + n$$

subject to  $x \geq 0$ .

Substituting the expression of price  $P$  given in Lemma 1, the above can be rewritten as

$$\max -kx(x+u-s)\alpha_m\sigma^2 - \frac{k^2\alpha_f}{2}x^2\sigma^2 + n$$

Taking first order condition, we get

$$x = -\frac{\alpha_m(u-s)}{2\alpha_m + k\alpha_f}$$

If  $u = -s$ , the optimal demand is  $x = \frac{2\alpha_ms}{2\alpha_m + k\alpha_f}$ . However, when  $u = s$ , the optimal demand is  $x = 0$ . Hence we can see that in the baseline case the manager does not have incentives to trade the asset if there is no negative demand shock.

By substituting  $x$  for  $s$ , we can see that the equilibrium price given  $x > 0$  is

$$P = d - (\alpha_m + k\alpha_f)x\sigma^2$$

Finally, the investors are willing to invest as long as

$$n(1 + \mathbf{E}[\frac{x(d + \epsilon - P)}{m + n}]) - n > 0 \quad (1)$$

This is satisfied given Assumption 1.

## 4.2 Interim Investment Case

Now let's look at the case where the investors do not observe  $d$  or  $u$ . We will show that there exists an equilibrium in which the fund manager buys overvalued assets in order to attract interim investments.

**Proposition 2.** *Given Assumption 1, the following is a Nash equilibrium:*

- When  $u = -s < 0$ , the manager chooses  $x = \frac{2\alpha_ms}{2\alpha_m + k\alpha_f}$ .
- When  $u = s$ , the manager chooses  $x = \frac{2\alpha_ms}{2\alpha_m + k\alpha_f}$  if  $d + 2\alpha_m\sigma^2 \leq H$ , and 0 otherwise.

- When  $P < H$ , the investors invest  $W$  in the fund if  $x = \frac{2\alpha_m s}{2\alpha_m + k\alpha_f}$ , and 0 otherwise.
- When  $P \geq H$ , the investors invest 0 in the fund.

It is clear that the asset will be undervalued if there is a negative demand shock ( $u = -s$ ), and that the asset will be overvalued if there is no demand shock and the fund manager submits a positive demand. The equilibrium implies that the fund manager will submit the same demand (unless  $d + \alpha_m s \sigma^2 > H$ ) no matter whether the asset is overvalued or undervalued. Moreover, in equilibrium, the investors will invest in the fund as long as  $P < H$ . To see why the equilibrium holds, let's start by looking at the case where the asset is undervalued ( $u = -s$ ).

When the asset is undervalued, a fund manager who is interested in maximizing investor wealth, will submit an optimal demand  $x = \frac{2\alpha_m s}{2\alpha_m + k\alpha_f}$  as shown in Proposition 1 in the baseline model. Because the investors are already going to invest in the fund, it is easy to see that the fund manager does not want to deviate from the optimal demand. Hence the demand of the fund manager is the same as in the baseline model.

Now suppose that there is a positive demand shock. Because there is no information asymmetry between the fund manager and the market maker, it is optimal for him not to trade in the baseline case. However, the investors will not invest in the fund after observing that the manager did not trade. Hence, the fund manager has incentives to pretend that the asset is undervalued in order to attract investments. Because the "correct" demand is  $x = \frac{2\alpha_m s}{2\alpha_m + k\alpha_f}$  if the asset really is undervalued, the fund manager must choose  $x = \frac{2\alpha_m s}{2\alpha_m + k\alpha_f}$  if he was to pretend. Certainly, the asset will be overvalued in this case.

If  $u = s$  and the manager submits  $x = \frac{2\alpha_m s}{2\alpha_m + k\alpha_f}$ , the price of the asset is  $P = d + \alpha_m x \sigma^2$ . The corresponding fundamental value that the asset would have if  $u = -s$  is  $P + (\alpha_m + k\alpha_f)x\sigma^2$ , which can be simplified to  $d + 2\alpha_m s \sigma^2$ . If  $d + 2\alpha_m s \sigma^2 > H$ , the investors would clearly infer that the asset is overvalued, since the fundamental value of the asset cannot exceed  $H$ . Hence the fund manager does not want to buy the asset if  $d + 2\alpha_m s \sigma^2 > H$ .

Suppose  $d + 2\alpha_m s \sigma^2 \leq H$ . In this case, the fund manager maximizes his expected utility

$$\mathbf{E}\{U(k[x(d + \epsilon - P) + m + n(x)])|u, d\}$$

where

$$n(x) = \begin{cases} W & \text{if } x = \frac{2\alpha_m s}{2\alpha_m + k\alpha_f} \\ 0 & \text{otherwise} \end{cases}$$

It is obvious that  $\mathbf{E}(U|x = 0) > \mathbf{E}(U|x > 0)$  for  $x \neq \frac{2\alpha_m s}{2\alpha_m + k\alpha_f}$ . Hence we only need to compare  $\mathbf{E}(U|x = 0)$  and  $\mathbf{E}(U|x = \frac{2\alpha_m s}{2\alpha_m + k\alpha_f})$ . We have

$$\begin{aligned} & \mathbf{E}\{U(k[x(d + \epsilon - P) + m + I(x)])|x = \frac{2\alpha_m s}{2\alpha_m + k\alpha_f}\} \\ &= \mathbf{E}\{-\exp(-k\alpha_f[x(d + \epsilon - P) + m + W])\} \\ &= -\exp\left(-k\alpha_f[x(-\alpha_m x \sigma^2) - \frac{k\alpha_f}{2}x^2 \sigma^2 + m + W]\right) \\ &= -\exp\left(-k\alpha_f\left[-\frac{2\alpha_m^2 s^2 \sigma^2}{2\alpha_m + k\alpha_f} + m + W\right]\right) \end{aligned}$$

Hence we have  $\mathbf{E}(U|x = \frac{2\alpha_m s}{2\alpha_m + k\alpha_f}) > \mathbf{E}(U|x = 0)$  if and only if

$$W > \frac{2\alpha_m^2 s^2 \sigma^2}{2\alpha_m + k\alpha_f} \quad (2)$$

Assumption 1 ensures that inequality 2 is satisfied.

Now let's look at the investors. First of all, we need to show that the investors cannot infer whether the asset is undervalued or overvalued from the price and the manager's demand. Given that  $x = \frac{2\alpha_m s}{2\alpha_m + k\alpha_f}$ , the fundamental component of the asset,  $d$ , is  $P - \alpha_m x \sigma^2$  if the asset is overvalued ( $u = s$ ), or  $P + (\alpha_m + k\alpha_f)x \sigma^2$  if the asset is undervalued ( $u = -s$ ). Recall that the investors cannot observe  $d$  nor  $u$ . The investors' posterior belief about the distribution of  $d$  is  $\mathbf{P}(d = P - \alpha_m x \sigma^2|x, P) = \frac{\frac{1}{2} \cdot \frac{1}{H-L}}{\frac{1}{2} \cdot \frac{1}{H-L} + \frac{1}{2} \cdot \frac{1}{H-L}} = \frac{1}{2}$  and  $\mathbf{P}(d = P + (\alpha_m + k\alpha_f)x \sigma^2|x, P) =$

$1 - \mathbf{P}(d = P - \alpha_m x \sigma^2 | x, P) = \frac{1}{2}$ . Hence the investors cannot infer whether the asset is overvalued or undervalued.

As we have mentioned before, the investors are willing to invest in the fund if and only if the expected return of the investment after management fee is greater than zero, i.e. they will choose  $n \geq 0$  to maximize

$$n(1 + \mathbf{E}[\frac{x(d + \epsilon - P)}{m + n}]) - n \quad (3)$$

where  $n$  denotes the amount of new investment.

First order condition implies that

$$(m + n)^2 = \frac{m(1 - k)\mathbf{E}[x(d + \epsilon - P)]}{k} = \frac{1}{2}(1 - k)\alpha_f x^2 \sigma^2 m$$

Substitute for  $x$ , we have

$$W < n^* = \frac{2\alpha_m s \sigma}{2\alpha_m + k\alpha_f} \sqrt{\frac{(1 - k)\alpha_f m}{2}} - m \quad (4)$$

This is a maximum because the second order derivative of expression 3 is negative. The investors' expected return will exceed the management fee as long as  $n^* > 0$ , i.e. the investors are willing to invest a positive amount in the fund. Moreover, we focus on the case where  $W < n^*$  and the investors can only invest up to  $W$  in the fund at  $t = 1$ , in order to avoid additional complications. Therefore inequality 2 ensures that  $n^* > 0$ .

**Proposition 3.** *Given the equilibrium in Proposition 2, the equilibrium prices are described as follows:*

$$P = \begin{cases} d - \frac{2(\alpha_m + k\alpha_f)}{2\alpha_m + k\alpha_f} \alpha_m \sigma^2 s & \text{if } u = -s \\ d + \frac{2\alpha_m^2}{2\alpha_m + k\alpha_f} \sigma^2 s & \text{if } u = s \text{ and } d \leq H - 2\alpha_m s \sigma^2 \\ d & \text{otherwise} \end{cases}$$

We can see that the price of the asset is always greater or equal to the price in the benchmark case with symmetric information. When  $u = s$  and  $d \leq H - 2\alpha_m s \sigma^2$ , the asset price is strictly greater than in the benchmark case. This is because the fund manager buys the overvalued asset in order to attract future investments. Hence we can see that the ex-ante expected price of the asset is inflated due to such incentives.

### 4.3 Equilibrium Discussion

Note that there is no separating equilibrium given that the parameter restrictions in Proposition 2 are satisfied. This is because it is always optimal for the manager to buy if the asset is undervalued, regardless of interim cash flows from the investors. When the asset is overvalued, the manager simply decides whether to mimic or not, and hence pooling and separating equilibria cannot coexist under the same parameter values. However, there may exist mixed-strategy equilibria in which the manager buys the overvalued asset with a certain probability. We do not consider them in this paper as they do not provide additional insights.

### 4.4 Summary

We have shown that, given certain parameter values, there exists an equilibrium where the fund manager buys overvalued assets in order to attract interim investments. Given management fees, the investors are only willing to invest in the fund if they expect the fund to make enough profits in the future. Because the investors can observe the portfolio of the fund, they would not invest if they discover that the fund manager did not trade. Anticipating this, the fund manager has incentives to submit a positive demand, even when the asset is overvalued, in order to pretend that the fund has the potential to increase in NAV. Because the expected fund return is still greater than the management fees, the investors are willing to invest in the fund after observing a buy order from the manager. Hence the fund manager would buy overvalued assets at the expense of investors due to potential interim investments.

We can see that the size of the interim investment,  $n$ , is increasing with both  $s$ , the size



of the negative demand shock, and  $\sigma$ , the volatility of the random component of the risky asset's payoff. This is intuitive because both  $s$  and  $\sigma$  are positively related to the extent of undervaluation of the asset in presence of a negative demand shock. The more undervalued the asset is, the higher return the fund is expected to achieve, hence the larger investment the investors are willing to make.

In equilibrium, the manager will almost always trade no matter what his private information about the asset's value is, although it is not optimal to trade when the asset is overvalued from the investor's perspective. This can partly explain why we observe large volume of trading in the delegated portfolio management industry, as by trading the asset the fund manager sends a signal to the investors that about the potential profitability of the fund.

The equilibrium is only valid with certain restrictions on parameters. In particular, we need the interim investment to be large enough that the manager has enough incentives to trade the overvalued asset. Because the investors will choose an optimal level of investment according to the fraction of management fee, we need this optimal level of investment to be greater than the trading loss, which was expressed in Assumption 1, i.e.  $\frac{2\alpha_m s}{2\alpha_m + k\alpha_f} (\sqrt{\frac{\alpha_f m}{2}} - \frac{\alpha_m s \sigma}{2}) > m$ . As said before, this inequality will be satisfied with a large  $\alpha_f$  and a small  $k$ . This is reasonable because, when we assume CARA utility, the fund manager is going to submit very large orders given that he is only compensated a fraction  $k$  of the assets he trades. The large demand from the manager will reduce the undervaluation of the asset (when  $u < 0$ ) and hence the expected return of the fund, leaving the investors with no incentives to invest. Therefore, by assuming that  $\alpha_f$  is large relative to  $\alpha_m$ , we can expect the fund manager submits more "reasonable" orders and the equilibrium will hold.

## 5 Conclusion

This paper presents an equilibrium in which the fund manager always buys the asset regardless of whether the asset is overvalued or undervalued. The incentive of trading is to attract the cash flows from investors who have no direct access to the market so that the fund manager's

final compensation will be increased. Due to the non-informative trading behavior of the fund manager, investors cannot learn anything from the equilibrium price so that they will make the investment as long as the expected return based on their belief exceeds the risk free rate. This would lead to higher risky asset prices and a higher-than-optimal proportion of investment in risky assets in the active fund management industry

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